

ANALYSIS OF HOT DENSE PLASMAS AND
CONSIDERATION OF STARK BROADENING
THEORY APPLIED TO TRANSITIONS INVOLVING
CONTINUUM RADIATOR WAVEFUNCTIONS

By

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Causa latet, vis est notissima.

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Abstract of Dissertation Presented to the Graduate School
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The use of Stark broadened lineshapes in the analysis of hot dense plasmas is studied. Analysis of emission spectra from experiments performed at the Laboratory for Laser Energetics at the University of Rochester including the effects of ion dynamics and opacity is presented. A method for the analysis of inhomogeneous absorbing regions is explored. The importance of auto-ionizing satellites in these analyses motivates an examination of the application of Stark broadening theory to those transitions involving continuum radiator wavefunctions.

Transitions involving continuum radiator wavefunctions have become essential diagnostic features of x-ray spectra from inertial confinement fusion experiments. Analysis of emission spectra including argon K-shell transitions and auto-ionizing satellites is presented, with emphasis on the intertwined effects of ion dynamics and opacity. Also, a method of analysis of absorption spectra is discussed, and a model for absorption by a non-homogeneous plasma is explored.

The application of Stark broadening theory to transitions involving continuum radiator wavefunctions is studied. Techniques and approximations appropriate to this application are developed and contrasted with those used in the analysis of transitions involving only bound radiator wavefunctions. For instance, for transitions among bound states the resolvent is frequently calculated by simple tetradic inversion. For transitions involving continuum radiator wavefunctions, the calculation is accomplished by discretizing the relevant continua and inverting the resulting tetradic. For transitions among bound states, the interaction of the radiator with the plasma ions is often approximated by truncating a multipole expansion. Here, I introduce an approximation appropriate for transitions involving continuum radiator wavefunctions. This approximation retains the calculational simplicity of the ion microfield formulation of Stark broadening theory, reduces to the dipole approximation for tightly bound states, and accounts for the spatial variation in the plasma ion microfield. These techniques and approximations are illustrated by the use of model systems, constructed to highlight the new features introduced in this work while avoiding the obscuring computational complications of actual atomic physics. The effect of continuum states on some transitions of current interest will be discussed.

CHAPTER 1 INTRODUCTION

The radiation emitted by ions embedded in a plasma from transitions among bound states differs significantly from that emitted by an isolated radiator. The plasma environment, interacting with each radiator through the combined electric field of the constituents of the plasma, changes the level structure of each radiator. The amount and character of that change differs for each radiator. The distribution of change, and thus the emitted spectrum, is dependent on a few plasma parameters, such as the density and temperature of the ions, and electrons. This dissertation examines the process of inferring plasma conditions from the spectrum emitted by embedded radiators, and discusses the application of this process to transitions between bound and unbound states.

At the outset, it is appropriate to parse the title of this work, as it contains some terms of art: "The Analysis of Hot Dense Plasmas and Consideration of Stark Broadening Theory Applied to Transitions Involving Continuum Radiator Wavefunctions". The subject matter of this dissertation is thus divided into two broad categories.

For the purpose of this work "dense plasmas" are those plasmas where the electron and ion number densities are sufficiently high so that plasma perturbations cause line widths greater than the sum of both the natural width and the Doppler width of the line. So, "Hot" can not be too hot, but rather indicates that the plasma electrons are non-degenerate. "Hot dense plasmas" are

then those non-degenerate plasmas for which Stark broadening is the dominant cause of the shape of the observed line profiles. "Analysis" here indicates the comparison of spectral data from such plasmas with calculated spectra composed of Stark broadened line profiles. The shapes of the calculated spectra depend on the temperature, density and size of the plasma. Those parameters used to calculate the spectrum which best fits the data provide the inferences for the parameters which characterize the plasma. This dissertation discusses two distinct analytical methods: the analysis of data from laser-imploded gas filled microballoons using the emitted line spectra, and the analysis of the detailed structure of absorbing regions.

"Stark broadening" refers to the effect that plasma microfields have on the energy level structure of an immersed radiator and the resulting broadening of the spectral line profile. "Transitions involving continuum radiator wavefunctions" are those where either the initial state and/or the final state of the transition considered has at least one unbound electron. Stark broadening theory is a mature discipline^{1,2,3,4,5,6} which for decades has been applied successfully to analyze plasma broadened transitions involving only bound radiator wave functions. The application of that theory to transitions involving unbound- as well as bound- radiator electrons is the subject matter of the final part of this dissertation.

This Introduction will give an overview of one method of producing hot dense plasmas and one method of recording time-resolved x-ray spectra. (While this is not an experimental dissertation, it is important to discuss some details of the experiments which produce the plasmas of interest.) Also, sample spectra will be presented for the purpose of establishing the nomenclature that will be used to denote various spectral features. With this context established,

the subsequent chapters of this dissertation will discuss Stark broadening theory, analysis of emission and absorption spectra, and the application of Stark broadening theory to diagnostically useful autoionizing satellites; the latter necessitates the development of some different approaches to the calculation of their lineshapes.

Typical of the plasmas addressed in this dissertation are those created using the process of laser-driven implosion⁷, illustrated in Figure 1.

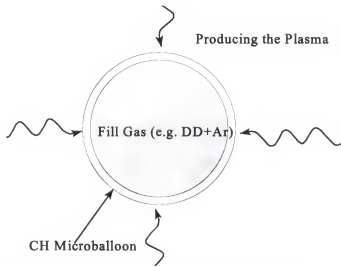


Figure 1. Laser-driven implosion. In direct drive laser-driven implosions, as currently implemented at the Laboratory for Laser Energetics at the University of Rochester, a spherical microballoon of approximately 1mm diameter and $15\mu\text{m}$ shell thickness, filled with a core gas, or fuel, is imploded by the reaction force to the ablation of shell material by the laser.

In this process, the laser symmetrically illuminates the spherical microballoon, depositing energy, mainly through inverse bremsstrahlung, in the exterior region of the shell material. The material in this region ablates, and the reaction force drives the remaining part of the shell inwards, compressing the fuel gas to densities many times solid density and temperatures near those of the interior of the Sun⁸.

The preceding Panglossian paragraph discusses an ideal seldom approached in actual experiment. There are many experimental realities which have so far prevented implosions from achieving the compressions predicted by spherically symmetric theories. For example, an important nemesis is the Rayleigh-Taylor instability.^{9,10,11} This hydrodynamic instability causes any non-uniformities in an interface where a less-dense fluid is pushing against (*i.e.*, accelerating) a more-dense fluid to grow exponentially. There are two such interfaces in an ablatively driven implosion, the interface between the ablating material and the remnant of the shell, and the interface between the fuel gas and the interior of the shell. The Rayleigh-Taylor instability can under some circumstances lead to shell break-up before the implosion is complete, limiting the convergence of the implosion. Any non-uniformities in the illumination of the shell by the laser can seed this instability even in the absence of shell non-uniformities.

The time-resolved x-ray spectra analyzed later in the dissertation are measured using crystal spectrometers attached to streak cameras. The geometry of these instruments is illustrated in Figure 2. In the spectral range of the Ar K-shell transitions which will be a focus of this dissertation (approximately 3-4 keV), these instruments operate with a resolution $E/\Delta E \approx 1000$, with a time resolution of approximately 40ps. As the time during which diagnostically useful x-rays are emitted from the plasma is approximately half a nanosecond, this time resolution suffices for the analysis of a coarse time history of that part of the implosion. As will be seen in the next chapter, the Stark broadened line widths for the lines of interest are on the order of tens of eV for the lowest densities considered, and thus the instrumental width associated with the resolution of the spectrometer does not prevent Stark broadening analysis.

Time-resolved X-ray Spectroscopy

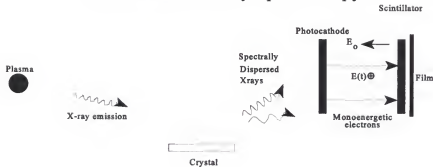


Figure 2. Time-resolved Spectrometry. As the incident x-rays enter the instrument, they are spectrally dispersed by a crystal. The dispersed x-rays then strike a photocathode, which produces approximately monoenergetic electrons at the locations of the impinging x-rays. These electrons are then pulled by a constant electric field towards the scintillator. In the drift region, a time varying electric field in a direction perpendicular to both the bias causing the drift and the spectral dispersion direction causes electrons emitted at different times to be deflected through different angles. The electrons then impact on a scintillator, behind which is the film on which the spectrum is recorded.

A typical set of time-resolved spectra, occasionally referred to as lineouts, from an emission experiment is shown in Figure 3. The details of target design, fill gas, and laser conditions which resulted in this data differ slightly from the direct-drive process discussed above. For the present purposes, it suffices to say that the fill gas consisted of a trace amount of Ar in D_2 , and that the line emission seen in the spectra is from the K-shell ions of Ar and some L-shell satellites. The labels in the figure are abbreviations for common designations of the lines. The general pattern for these designations is [element] [Rydberg series]-[Greek letter indicating the upper state]. The radiator in Figure 3 is Argon. One of the two prominent Rydberg series in the spectrum is the Lyman

series in Hydrogen-like Argon (known variously as Ar^{+17} and Ar XVIII). The transitions in this series are $n\ell \rightarrow 1s$ where n is 2 for the α line, 3 for the β line, and so forth. Thus, the line labeled Ly- α in Figure 3, located at 3320eV is the 2 to 1 transition in Hydrogen-like Ar. The other prominent Rydberg series in Figure 3 is the $1sn\ell \rightarrow 1s^2$ series in Helium-like Argon, denoted the Argon Helium series. For instance, the Argon Helium- α (abbreviated He- α), the $1s2\ell \rightarrow 1s^2$ transition is located at 3130eV.

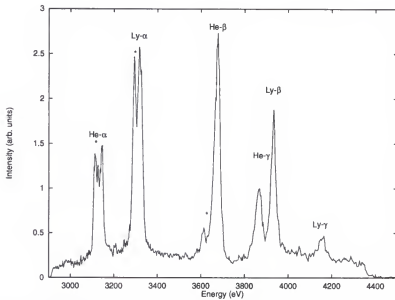


Figure 3: A Typical Emission Spectrum. This spectrum shows the diagnostically important Argon features in the spectral range from 2900-4500eV. The labels are explained in the accompanying text. The stars indicate the position of some prominent satellite transitions.

Figure 4 displays a typical absorption feature, from a target where the fill gas was pure Ar. In this case, the emission from the core of the target has been attenuated when passing through a relatively cool and dense region. The absorption feature is composed of absorption lineshapes from the auto-ionizing satellites of the Argon He-alpha resonance line. A satellite line is one in which

the active electron makes the same transition as a resonance line, but where there are other excited spectator electrons. Thus, a Li-like satellite of the He- α would be a $1s2\ell n\ell' \rightarrow 1s^2 n\ell''$, for $n \geq 2$, a 2 to 1 transition in Li-like Ar with a spectator in the n th energy level. Usually, the primary effect of the spectator electron is to screen the nuclear charge as seen by the active electron, thus reducing the energy of the transition. The absorption feature in Figure 4 is composed of many of these satellites, separated in energy due to differing numbers of spectator electrons.

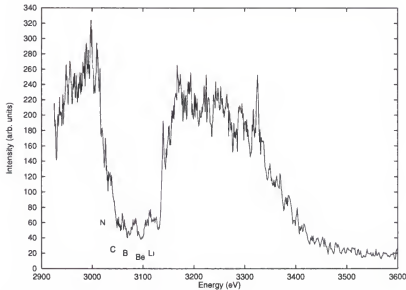


Figure 4. A Typical Absorption Spectrum. This spectrum shows a typical absorption feature from a target with a pure Ar fill gas. The labels indicate the location of the absorption features due to the differing ionization stages of Ar.

Table 1 gives the name and location of the Argon lines that are used in the subsequent chapters. The locations given are the centers-of-gravity of the line, as each line consists of many transitions from and to differing angular momentum states.

Table 1: Energy locations of several lines in the K- and L-shell ions of Ar. Location is obtained by weighting the individual transitions which compose the lines by line strength and the statistical weight of the initial state.

Transition	Energy (eV)
Ly- α	3320
Ly- β	3935
Ly- γ	4150
He- α	3130
He- β	3680
He- γ	3880
He- δ	3965
He-l sat. of Ly- α , spec. in $n = 2$	3290
He-l sat. of Ly- α , spec. in $n = 3$	3315
He-l sat. of Ly- β , spec. in $n = 2$	3870
He-l sat. of Ly- β , spec. in $n = 3$	3915
Li-l sat. of He- β , spec. in $n = 2$	3620
Li-l sat. of He- β , spec. in $n = 3$	3660
Li-l sat. of He- α	3110
Be-l sat. of He- α	3085
B-l sat. of He- α	3060
C-l sat. of He- α	3035
N-l sat. of He- α	3015
O-l sat. of He- α	2990
F-l sat. of He- α	2975

Having given an overview of the experimental technique, and hopefully having whetted the reader's appetite, this dissertation introduces Stark broadening theory, its use in the analysis of plasmas, and considerations of its application

to transitions involving continuum radiator wavefunction such as the satellites that shall form an important part of the analysis procedures.

CHAPTER 2 STARK BROADENING THEORY

The formal development of Stark broadening theory is detailed in many places in the literature^{2,12,13}, and is presented here not with any pretense of novelty, but with the intention of establishing the theory for those unfamiliar with it and establishing the notation to be used throughout this work. The aim in Stark broadening theory is to calculate the broadened lineshapes emitted by radiators immersed in a plasma. The plasma environment perturbs the energy level structure of the radiator, giving rise to a lineshape characteristic of the temperature and density of the plasma, thus providing a non-intrusive diagnostic of plasma conditions.

The model for the plasma to be used in this work assumes that the radiating ions constitute a small fraction of the total number of ions, and that they do not interact with each other. To develop the formalism, the plasma is modeled as a number of non-interacting cells each consisting of one radiating ion, perturbing ions, and unbound electrons. The electric dipole emission from the plasma is that from an ensemble of such cells.

The Hamiltonian for a cell in the plasma, H , is

$$H = H_r^0 + H_i^{0'} + H_e^0 + V_{i,r} + V_{e,r} + V_{i,e},$$

where H_r^0 is the Hamiltonian for an isolated radiator with nucleus of charge Z ,

$$H_r^0 = \sum_j \left[\frac{p_j^2}{2m} - \frac{Ze^2}{|\vec{r}_j|} + \sum_{k < j} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} \right].$$

$H_i^{0'}$ is the Hamiltonian for the perturbing ions, here taken as structureless point charges of mass M and charge χ :

$$H_i^{0'} = \sum_q \left[\frac{p_q^2}{2M} + \sum_{s < q} \frac{\chi^2 e^2}{|\vec{r}_j - \vec{r}_k|} \right].$$

The Hamiltonian for the plasma electrons, H_e^0 is

$$H_e^0 = \sum_u \left[\frac{p_u^2}{2m} - \sum_{s < u} \frac{e^2}{|\vec{r}_u - \vec{r}_s|} \right].$$

The interaction terms in the Hamiltonian, $V_{i,r}$, $V_{e,r}$, $V_{i,e}$ are given by:

$$\begin{aligned} V_{i,r}' &= \sum_q \left[\frac{Z\chi e^2}{|\vec{R}_q|} - \sum_j \frac{\chi_i e^2}{|\vec{r}_j - \vec{R}_q|} \right] \\ V_{e,r} &= \sum_u \left[-\frac{Ze^2}{|\vec{r}_u|} + \sum_j \frac{e^2}{|\vec{r}_j - \vec{r}_u|} \right] \\ V_{i,e} &= \sum_{q,u} \left[-\frac{\chi e^2}{|\vec{r}_u - \vec{R}_q|} \right]. \end{aligned}$$

Thus the subscripts r , i , and e respectively refer to the radiator, the perturbing ions and the perturbing electrons, and H_μ^0 is the Hamiltonian for an isolated μ subsystem, and $V_{\mu,\nu}$ are the coulombic potentials of interaction between the μ and ν subsystems. The origin of coordinates is located at the nucleus of the radiator, and the effects of radiator motion approximated by the appropriate Doppler convolution.[†]

[†] It is apparent that the use of a simple convolution with a gaussian to account for the effect of radiator motion is an approximation in which any correlation between the radiator's motion and the other broadening processes is ignored. The correlation is best illustrated by the fact that in the center of mass frame of the radiator, the plasma perturbers have an average velocity. The radiators moving fastest in the lab frame, those responsible for the wings of the Doppler gaussian, have a higher collision frequency with the plasma perturbers. The effect of this correlation is explored by Smith, *et al.*, (JQSRT **11**, 1547 (1971), JQSRT **11**, 1567 (1971)). Kilcrease (Ph.D. dissertation (University of Florida, 1991)) discusses the approximation as it relates to the current formalism. The question of the correlation between the broadening due to dynamic perturber ions and Doppler broadening is considered in Ref. 14. As a practical matter, for the cases considered here, the Doppler width is small

The terms in the Hamiltonian are conveniently regrouped as follows

$$\begin{aligned} H &= H_r + H_i' + H_e \\ H_r &= H_r^0 + V_{i,r}^{1'} + V_{e,r}^1 \\ H_i' &= H_i^{0'} + V_{i,r}^{0'} + V_{i,e} \\ H_e &= H_e^0 + V_{e,r}^0 \end{aligned}$$

where the monopole terms of the interaction (denoted by the superscript “0”) of the radiator and the plasma perturbors, which do not depend on the radiator electron coordinates have been separated from the rest of the interactions (denoted by the superscript “1”). In this form, neither H_i' nor H_e depend on the radiator electron coordinates, and the picture is one of the radiator interacting with a “bath” of plasma electrons and ions through the interactions $V_{i,r}^{1'}$, $V_{e,r}^1$.

Up to this point in the development of the formalism, the plasma Hamiltonian is that of a stationary radiating ion located at the origin of the coordinate system, consisting of a nucleus of charge Z and some radiator electrons, interacting with a bath of perturbing ions (structureless point charges with charge χ) and perturbing electrons, all interacting through bare Coulomb potentials. The difference in mass, and the resulting differences in typical velocities and time scales, between the plasma ions and electrons motivates the incorporation of the electron-ion interaction, $V_{i,e}$, into an effective Debye screened ion-ion interaction.¹⁵

enough compared to the other Stark widths that the detailed shape of the Doppler function is unimportant. The lines are primarily electron broadened, and the momentum transferred to the radiator by each electron collision is negligible, just the condition which Smith *et al.* found was sufficient to justify the use of a gaussian for the Doppler broadening.

The resulting Hamiltonian is

$$H = H_r + H_i + H_e$$

$$H_r = H_r^0 + V_{i,r}^1 + V_{e,r}^1$$

$$H_i = H_i^0 + V_{i,r}^0$$

$$H_e = H_e^0 + V_{e,r}^0,$$

where now the ions interact through a screened Coulomb potential. The radiator is perturbed by the bath of plasma electrons and ions.

Having established a model for the plasma, it is possible to calculate the lineshape due to electric dipole transitions among radiator states. It is appropriate to start by considering the expression for the total power emitted by a quantum system undergoing an electric dipole transition¹⁶ from state a to state b ,

$$P_{a \rightarrow b} = \frac{4\omega_{ab}^4}{3c^3} | \langle b | \vec{d} | a \rangle |^2.$$

$\hbar\omega_{ab}$ is energy of the transition, c the speed of light, and $\vec{d}$ is the dipole operator for the system. States a and b are eigenstates of the system Hamiltonian with energies E_a and E_b , respectively. The emission from the plasma is being modeled as that of an ensemble of non-interacting cells; hence, the power spectrum emitted by the plasma is obtained by averaging over the initial states a , weighted by the population of state a , ρ_a , and summing over the possible final states

$$P(\omega) = \sum_{a,b} \rho_a \frac{4\omega_{ab}^4}{3c^3} | \langle b | \vec{d} | a \rangle |^2 \delta(\omega - \omega_{ab}).$$

As the ω^4 term will vary little over the width of the lines considered here, it is convenient to define the lineshape as

$$I(\omega) = \sum_{a,b} \rho_a | \langle b | \vec{d} | a \rangle |^2 \delta(\omega - \omega_{ab}),$$

with the power given by

$$P(\omega) = \frac{4\omega^4}{3c^3} I(\omega).$$

It is instructive to consider the Fourier transform of $I(\omega)$, $\Phi(t)$:

$$\begin{aligned}\Phi(t) &= \int_{-\infty}^{\infty} d\omega e^{-i\omega t} I(\omega) \\ \Phi(t) &= \sum_{a,b} \rho_a e^{-i\omega_{ab}t} |< b|\vec{d}|a>|^2\end{aligned}$$

Thus, $\Phi(-t) = [\Phi(t)]^*$ and

$$I(\omega) = \frac{1}{\pi} \Re \int_0^{\infty} dt e^{i\omega t} \Phi(t).$$

Up to now the transform variable, t , has no explicit physical interpretation. Next, $\Phi(t)$ is shown to be the auto-correlation function for the dipole moment operator of the cell, in order that we might take advantage of the general properties of autocorrelation functions. The following development is a specific demonstration of a general principle, namely the Weiner-Khinchin Theorem¹⁷

Start by recalling that ρ_a , the population of the initial states of the system, was introduced in order to weight the averaging over initial states of the system. Let ρ be the equilibrium density matrix for the plasma with the screened potentials discussed above. As the present purpose is to calculate emission from transitions among radiator states, restrict ρ such that it is diagonal and non-zero only for the initial states of such transitions. Thus the effects of continuum radiation from the perturbing plasma, such as bremsstrahlung, are excluded from these calculations. Thus defined, insert this operator into a matrix element. Using the definition of ω_{ab} , write

$$\Phi(t) = \sum_{a,b} e^{\frac{-i(E_a - E_b)t}{\hbar}} |< b|\vec{d}|a> \cdot < a|\rho\vec{d}|b>|.$$

Noting that the energies E_a and E_b are the eigenvalues of the system Hamiltonian in states a and b respectively, aggregate all the terms into the matrix elements:

$$\Phi(t) = \sum_{a,b} \langle b | \vec{d} | a \rangle \cdot \langle a | e^{\frac{-iHt}{\hbar}} \rho \vec{d} e^{\frac{iHt}{\hbar}} | b \rangle.$$

The restricted definition of ρ allows the inclusion of all states in our sum over a , and a further restriction, namely that $\vec{d}$ should only be non-zero between initial and final states which define the problem (*i.e.*, the lineshape whose calculation is desired) allow the expression of the sum as a trace:

$$\begin{aligned} \Phi(t) &= \text{Tr}[\vec{d} \cdot e^{\frac{-iHt}{\hbar}} \rho \vec{d} e^{\frac{iHt}{\hbar}}] \\ &= \text{Tr}[\vec{d} \cdot \rho \vec{d}(t)], \end{aligned}$$

and recognize this as the dipole auto-correlation function for our system.

Having established a physical interpretation for $\Phi(t)$, it is possible take to advantage of a property of time autocorrelation functions, namely that they are smoothly decaying functions for large t . Associate a characteristic time for this decay, $\Delta t_{\frac{1}{2}}$ with the decay of $\Phi(t)$. Using a familiar property of Fourier transforms, relate this characteristic time to the half-width of the lineshape,

$$\Delta\omega_{\frac{1}{2}};$$

$$\Delta\omega_{\frac{1}{2}} \Delta t_{\frac{1}{2}} \approx 1.$$

Thus, by examination of experimentally observed lineshapes, one can deduce the characteristic time associated with the decay of the autocorrelation function. Comparing this decay time to other characteristic times in our system will lead to differing approximations for the interaction of the radiator with the plasma ions and electrons.

It is now instructive to follow a generalization of Smith's¹⁵ demonstration of this point, though while he used hydrogen in a proton-electron plasma of

density of $n_e = 3.6 \times 10^{17} \text{cm}^{-3}$ and a temperature of 2eV, this discussion will illustrate the point for a plasma of more current interest, the Lyman- α emission from a hydrogenic Ar radiator in an Ar plasma of electron density $n_e = 1 \times 10^{24} \text{cm}^{-3}$ and a temperature of 1keV. The full-width at half maximum of this line is approximately 7eV. So, the time scale of the decay of the autocorrelation function is $\Delta t_{\frac{1}{2}} = 10^{-16} \text{sec}$. As the monopole terms in the interactions of the radiator with the plasma constituents has been included in the zero-order Hamiltonians for those subsystems, the first term treated as a perturbation in the interaction of the radiator with the plasma is the dipole-field interaction. This discussion proceeds with an examination of the changes in the electric fields at the radiator due to the ions and the electrons during a period time $\Delta t_{\frac{1}{2}}$.

A particle traveling at the thermal velocity, $v_{av} = \sqrt{3kT/m}$, will move a distance $\delta r_{av} = v_{av} \Delta t_{\frac{1}{2}}$ during the time of interest. The resulting change in the magnitude of the electric field at the radiator due to these particles (with charge χ) can be approximated as follows:

$$\begin{aligned} \vec{E} &= \chi e \sum_j \frac{\vec{r}_j}{|\vec{r}_j|^3} \\ \delta E &= \chi e \sum_j \delta(r_j^{-2}) = -2\chi e \sum_j \frac{(\delta r_j)}{r_j^3} \\ &\approx -2\chi e \left(\frac{\delta r_{av}}{r_{av}} \right) \sum_j \frac{1}{r_j^2} \approx - \left(\frac{\delta r_{av}}{r_{av}} \right) (2E). \end{aligned}$$

Thus, the fractional change in the magnitude of the electric field at the radiator due to a group of particles with mass m is given by

$$\left| \frac{\delta E}{E} \right| = 2\sqrt{3kT/m} \sqrt[3]{4\pi n/3} \Delta t_{\frac{1}{2}}$$

where n is the number density of these particles. For the electrons in our typical plasma, this ratio is 74.1, while for the Ar ions, this ratio is 0.1. (A

cautionary note in passing: this ratio is approximately 1 if the perturbors are protons rather than Ar ions.) Thus, the electric field due to the plasma electrons changes substantially during the time of interest, while the ion microfield changes fractionally. Therein lies the motivation for the static ion approximation, in which the perturbing ions are treated as static, and effecting the radiator through their combined electric field. This field can be thought of as splitting the energy levels of the radiator, and it is these split levels which are further perturbed by the dynamic electrons. The probability that the radiator experiences a certain value of the field, the ion microfield distribution function, is well understood, and can be calculated in a number of approximations¹⁸.

We have shown that the lineshape is given by the Fourier transform of the radiator's dipole-dipole time autocorrelation function:

$$I(\omega) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{rei}[\vec{d} \cdot \vec{d}(t) \rho_{rei}]$$

It is convenient to use the Liouville space formalism to treat the time development of the system¹, with the Liouville operator L defined by

$$L\Omega = [H, \Omega] \quad , \quad \vec{d}(t) = e^{\frac{-iLt}{\hbar}} \vec{d}.$$

In the static ion approximation, the evolution of the dipole moment of the radiator does not depend on the dynamics of the ion microfield for the time scales of interest. This approximation is reflected in Liouville operator,

$$\vec{d}(t) = e^{\frac{-iL_{er}t}{\hbar}} \vec{d}$$

The equilibrium density matrix operator, ρ , is approximated as the product of the density matrix for the plasma ions and that for the radiator and the plasma electrons,

$$\rho = \rho_i \rho_{re}.$$

This factorization approximation neglects the effects of V_{ir} and V_{ie} on the initial distribution of population, though the effect of V_{ie} is approximated through the use of the screened potentials. Further discussion of this approximation can be found in Ref. 19.

The task remains to calculate

$$I(\omega) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{rei} [\vec{d} \cdot e^{-\frac{iL_{er}t}{\hbar}} \vec{d} \rho_{re} \rho_i].$$

Recall that $H_{e,r} = H_e + H_r$, and thus L_{er} , depends on the perturbing ion coordinates through $V_{i,r}^1$. If $V_{i,r}$ depended on, or approximately depended on, the ion coordinates only through their combined electric field at the radiator,

$$\vec{E}_i = \sum_{\text{pert. ions}} \frac{-\chi_{pe}}{|\vec{R}_{\text{ion}}|^2},$$

then the introduction of the static ion microfield distribution is natural, and the calculation proceeds as detailed in the literature² and summarized below. Those cases in which the interaction of the radiator and the ions cannot be approximated by truncating the multipole expansion of the interaction after the dipole term are considered in Chapter 5.

In the dipole approximation, the interaction of the radiator with the ion microfield, $V_{i,r}$, is given by $\vec{E}_i \cdot \vec{D}$, where $\vec{D}$ is the dipole moment of the radiator, and $\vec{E}_i$ is the microfield at the position of the radiator due to the plasma ions, an operator which depends on the positions of the plasma ions. In the following, the ion coordinate dependence in the expression for the lineshape is explicitly notated.

$$I(\omega) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{rei} [\vec{d} \cdot e^{-\frac{iL_{er}(\{R_i\})t}{\hbar}} \vec{d} \rho_{re} \rho_i]$$

Using a delta function, we formally replace the operator $\vec{E}(\{\vec{R}_i\})$ with a constant vector, $\vec{E}$, at the price of eventually having to evaluate the thermal average of the delta function.

$$\begin{aligned} I(\omega) &= \frac{1}{\pi} \int d\vec{E} \delta(\vec{E} - \vec{E}(\{\vec{R}_i\})) \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot e^{-\frac{iL_{er}(\vec{E}(\{\vec{R}_i\}))t}{\hbar}} \vec{d} \rho_{re} \rho_i] \\ &= \frac{1}{\pi} \int d\vec{E} \text{Tr}_i \rho_i \delta(\vec{E} - \vec{E}(\{\vec{R}_i\})) \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot e^{-\frac{iL_{er}(\vec{E})t}{\hbar}} \vec{d} \rho_{re}] \end{aligned}$$

Now, all the ion coordinate dependence is in the ion microfield distribution function, $Q(\vec{E})$:

$$Q(\vec{E}) = \text{Tr}_i \rho_i \delta(\vec{E} - \vec{E}(\{\vec{R}_i\})),$$

and the lineshape problem is separated into calculating the probability that the radiator experiences a certain ion microfield, and then calculating the electron-perturbed lineshape emitted by a radiator experiencing that microfield:

$$\begin{aligned} I(\omega) &= \int d\vec{E} Q(\vec{E}) J(\omega; \vec{E}) \\ Q(\vec{E}) &= \text{Tr}_i \rho_i \delta(\vec{E} - \vec{E}(\{\vec{R}_i\})) \\ J(\omega; \vec{E}) &= \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot e^{-\frac{iL_{er}(\vec{E})t}{\hbar}} \vec{d} \rho_{re}] \end{aligned}$$

Calculation of the ion microfield distribution function has received significant attention¹⁸. The calculations here will use the APEX method of Iglesias, Lebowitz, and MacGowan, which has been shown to reliably reproduce the results of Hooper^{20,21} for low coupling, and the results of Molecular Dynamics simulations for higher coupling, and is considerably easier to implement than either of the other two approaches²². The calculation of $J(\omega; \vec{E})$, the single-ion-microfield electron-broadened lineshape in the relaxation approximation is straightforward, and available in exhaustive detail in the literature.^{2,3,4,6,15,23,24,13} Here, the calculation is sketched.

Performing the integration over time, assured of convergence by the finite natural lifetime of the upper level of the transition, an expression for $J(\omega; \vec{E})$ is obtained in terms of the inverse of a sum of Liouville operators:

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \text{Tr}_{r,e} \left[\vec{d} \cdot \frac{1}{\omega - L_{er}} \vec{d} \rho_{r,e} \right].$$

Approximating the initial density matrix $\rho_{e,r}$ as the product of the density matrices for the two subsystems, we write

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \text{Tr}_r \left[\vec{d} \cdot \text{Tr}_e \left[\rho_e \frac{1}{\omega - L_{er}} \right] \vec{d} \right].$$

The approximations inherent in the factorization of the density matrix have been discussed by Whalen.²⁵

The trace over plasma electron coordinates is approximated to second order in the radiator-plasma electron interaction by means of an expansion detailed in Ref. 23. The result is that

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \text{Tr}_r \rho_r \left[\vec{d} \cdot R(\omega; \vec{E}) \vec{d} \right],$$

with the resolvent, $R(\omega; \vec{E})$, given by

$$R(\omega; \vec{E}) = \frac{1}{\omega - L_r^0 - L_{ir}^1(\vec{E}) - M(\omega)}.$$

The electron broadening operator, $M(\omega)$ is related to the time autocorrelation function of the electric field at the radiator due to the plasma electrons.²³ In the quantum mechanical relaxation theory, using the dipole approximation to the radiator-electron interaction and keeping terms to second order in the interaction,

$$M(\omega) = \text{Tr}_e \left[\rho_e L_{er} \frac{1}{\omega - L_r^0} L_{er} \right].$$

With a sketch of the essential framework of Stark broadening theory thus introduced, the next two chapters illustrate the use of the lineshapes in the analysis of hot dense plasmas.

CHAPTER 3 THE ANALYSIS OF HOT DENSE PLASMAS

Introduction

X-ray emission spectroscopy has been successful in probing the hot core of laser-driven implosions^{26,5}. This spectroscopy is valuable as a means of diagnosing ICF plasmas, by means of mixing trace amounts of appropriately chosen high-Z gases into the hydrogen-isotope fuel such that, under the expected conditions of the implosion, the average dopant ion retains only a few electrons^{27,28,29,30,31}. The amount of dopant is ideally chosen to be sufficiently small to assure that the implosion dynamics are not appreciably effected by the dopant, but not so small as to render the diagnostic emission unobservable. Balancing these two considerations can lead to plasma compositions where a preponderance of the light fuel nuclei makes the static ion approximation discussed in the last chapter inappropriate in the calculation of Stark broadened line profiles. Here, this issue is examined by analyzing spectra from a series of implosion experiments where the dopant concentration was systematically altered. This chapter concentrates on an assessment of the individual and combined effects of ion motion and opacity on the lineshape and their impact on plasma diagnostics.

The next section presents the formalisms underlying the calculation of the temperature- and density-sensitive theoretical spectra. These were used to infer plasma conditions by comparison with the experimental spectra. Then,

time-resolved Ar emission spectra from Ar-doped implosions performed at the Laboratory for Laser Energetics at the University of Rochester are presented and analyzed. Based on the results of this analysis, conclusions and suggestions are presented where future theoretical and experimental work might prove fruitful.

Calculation of Theoretical Spectra

An essential idea motivates the use of Stark broadened line profiles to diagnose hot, dense plasmas: the line profiles themselves are strongly density dependent for the expected plasma conditions, yet are relatively insensitive to temperature. Thus, the detailed shape of the individual line profiles in the spectrum can be analyzed to yield an inference of plasma electron density. The ratios of the intensities of the lines in the spectrum are both temperature and density sensitive. Combined with the density dependence of the lineshapes, they can yield an inference of the plasma temperature. Our goal is to calculate temperature- and density-sensitive spectra with enough detail to model experimental data throughout the spectral region containing the Ar He- β ($1s3\ell \rightarrow 1s^2$), $-\gamma$ ($1s4\ell \rightarrow 1s^2$), $-\delta$ ($1s5\ell \rightarrow 1s^2$), and Ly- β ($3\ell \rightarrow 1s$) lines and their attendant satellites.

As a first approximation, the emitting region is modeled as a sphere of uniform density and temperature ($T_i = T_e$), with a diameter consistent with mass conservation, assuming that all of the deuterium is fully stripped. The ionization state of Ar is calculated using non-LTE atomic kinetics. The final section of this chapter discusses modifications of this model accounting for the existence of temperature and density gradients, and plasma induced line shifts motivated by the analysis of data presented here.

Stark Broadened Line Profiles Including the Effects of Ion Motion

In both the static^{13,12} and dynamic¹⁴ ion treatments of Stark broadening, the line shape can be written as

$$I(\omega) = \frac{4\omega^4}{3c^3} \int d\vec{E} Q(\vec{E}) J(\omega, \vec{E}),$$

where $Q(\vec{E})$ is the ion-microfield probability distribution function, here calculated in the APEX approximation²². In both cases, $J(\omega, \vec{E})$ can be written

$$J(\omega, \vec{E}) = -\frac{1}{\pi} \text{ImTr} [\vec{d} \cdot R(\omega, \vec{E}) \vec{d} \rho],$$

where the trace is over the relevant radiator states, $\vec{d}$ is the radiator electric dipole operator, and ρ is the density matrix of radiator states. In both the static and dynamic ion calculations, the resulting profiles are convolved with a temperature dependent gaussian profile to account for the Doppler effect. For the temperatures and transitions considered here, the Doppler FWHM is less than 2eV and this effect is included in all the results presented herein. The difference between the static and dynamic theory is in the form of the resolvent, $R(\omega, \vec{E})$. For the dynamic case, we implement the theory of Boercker, Iglesias and Dufty (BID),¹⁴ in which the resolvent is approximated as

$$R(\omega, \vec{E}) = \frac{G(\Delta\omega, \vec{E})}{1 + i\nu(\Delta\omega) \int d\vec{E} Q(\vec{E}) G(\Delta\omega, \vec{E})},$$

$$G(\Delta\omega, \vec{E}) = \frac{1}{\Delta\omega - \mathcal{L}_{i,r}^1(\vec{E}) - M(\Delta\omega) - i\nu(\Delta\omega)};$$

where $\Delta\omega = (\omega - \omega_{\mu,\nu})\delta_{\mu\mu'}\delta_{\nu\nu'}$, $\mathcal{L}_{i,r}^1(\vec{E})$ is the Liouville operator associated with the interaction of the ion microfield with the radiator's dipole, and $M(\Delta\omega)$ is the broadening tetradic due to the dynamic plasma electrons. All quantities with the exception of $\nu(\Delta\omega)$ are exactly those which need to be calculated in the static ion approximation using computational methods discussed in Ref. 12.

The only quantity not familiar from the static ion approximation is $\nu(\Delta\omega)$. In the limit $\nu(\Delta\omega) \rightarrow 0$, the result of the static ion approximation is recovered. Thus, $\nu(\Delta\omega)$ is a measure of the importance of ion motion effects on the lineshape. In Ref. 14, $\nu(\Delta\omega)$ is introduced as a parameter in a model for the memory operator in the evolution equation for radiator operators, fixed by the imposition of certain exact limits on the radiator momentum autocorrelation function (see Ref. 14, equation 2.11 and Section III for details). Following BID, the calculation discussed here approximates $\nu(\Delta\omega)$ as a real constant, ν , proportional to the self diffusion coefficient for the radiator species, D ,

$$D = \frac{1}{3} \int dt \langle \vec{v}(t) \cdot \vec{v}(0) \rangle.$$

The self diffusion coefficient was calculated using a “disconnected” approximation.¹⁴ For a fixed concentration of Ar in D₂, ν increases with increasing temperature and density. As the concentration of Ar increases, the increase in the reduced mass of the average radiator-perturbing ion pair leads to a reduction in ν , and thus to a decrease in the effect of ion dynamics on the lineshape.

The effects of ion dynamics are illustrated in Figure 5, where Stark broadened Ar He- β lineshapes are displayed for fixed temperature and electron density, and varying concentrations of Ar in D₂. As the concentration of Ar increases, the decreasing value of ν leads to a decreased ion-dynamic effect, and thus a more marked peak structure at or near line center. Changing the concentration of Ar in D₂ also modifies $Q(\vec{E})$, this primarily effects the wings and the separation between the peaks. The effects of ion dynamics are responsible for the large effect on the dip at line center. In general, the effect of ion dynamics is to broaden the line around the peaks, within one ion plasma frequency of each of the peaks. For lines with a sizable central dip, *e.g.*, the

β lines, the effect of ion dynamics is most obvious as a filling in of the dip. The effect of ion dynamics decreases with increasing Ar concentration, with the 100% Ar case strongly resembling the static result.

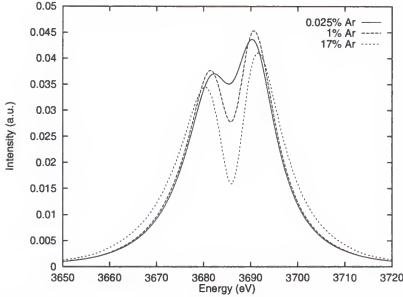


Figure 5: Area normalized dynamic Stark broadened Ar He- β lineshapes, calculated for 0.025% ($\nu = 3.37\text{eV}$), 1% ($\nu = 1.66\text{eV}$), and 17% Ar in D₂ ($\nu = 0.621\text{eV}$); $n_e = 5 \times 10^{23}\text{cm}^{-3}$, $kT = 900\text{eV}$.

Opacity Broadening

Having calculated the Stark broadened lineshapes including the effects of ion motion, it is necessary to include other effects which alter the observed line profile before making comparisons with the experimental data. In the optically thin approximation, the intrinsic lineshape (calculated as above) would be simply convolved with an instrumental response function. However, as the concentration of Ar increases one must account for opacity effects as well. The effects of opacity broadening, like those of ion dynamics, are most obvious near the peaks of the lineshape, though, unlike the effect of ion dynamics, opacity

can also distort the wings. At the peaks, the redistribution of photons is most readily apparent, as photons from the peaks of the line are redistributed into the wings where they have a better chance of escaping to be detected. For the small concentrations of Ar in D₂ in the experiments to be analyzed, the effect of opacity broadening is approximated using a modified slab opacity formula³². In this approximation, prior to convolution with the instrumental response function, the area-normalized observed lineshape is given by

$$I_{\text{obs}}(\omega) = AB(\omega)(1 - e^{-\tau_{\omega}}),$$

where A is a normalization constant, $B(\omega)$ is the Planckian distribution and τ_{ω} , the frequency dependent optical depth, is given by

$$\tau_{\omega} = \frac{\pi e^2}{mc} \bar{f} \hat{I}(\omega) N_l d;$$

where $\bar{f}$ is the effective oscillator strength for the transition of interest, the circumflex denotes area-normalization with respect to the frequency variable $\frac{\omega}{2\pi}$, N_l the number density of ions in the lower states of the transition, and d is the average chord length in the plasma. Figure 6 displays the effect of opacity broadening on the lineshape, comparing the Ar He- β line calculated in the optically thin, static ion approximation with the calculations which include first opacity and then both opacity and ion dynamics. When both ion dynamics and opacity effects are included, there is a corresponding decrease in the calculated optical depth at line peak, τ_0 : τ_0 , which was 0.66 for the static ion calculation in Figure 6, is reduced to 0.59 when the calculation includes ion dynamic effects. The peak structure of the observed lineshape is effected by both opacity broadening and ion motion, and the response of these two effects to changes in Ar concentration is competitive. Care must be taken to

account for both effects in lineshapes calculated for the purpose of analyzing experimental data.

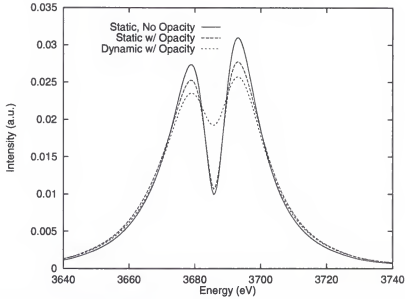


Figure 6. Ion dynamics and opacity effects on the area normalized Ar He- β lineshape, $n_e = 10 \times 10^{23} \text{cm}^{-3}$, $kT = 900 \text{eV}$, 0.25% Ar in D_2 , $d=25 \mu\text{m}$.

Non-LTE Relative Intensities, Radiative Transfer Effect

Having been calculated using the methods discussed above, the individual lineshapes must be combined with the correct relative intensities to form a theoretical spectrum. A non-LTE kinetics model³³ was used to calculate the relative intensities of the lines in the model spectrum. The model includes all the ionization stages in Ar from P-like Ar (Ar^{+3}) to the fully stripped ion (Ar^{+18}). For each ionization stage, the model includes the ground state configuration and several excited state configurations. The amount of detail in the model is greatest in the Li-, He- and H-like stages (the most populous ionization stages at the temperatures and densities of interest here), and doubly-excited

states are treated self-consistently with ground and singly-excited states. In particular, for Li-like Ar the model includes 918 energy levels, with sufficient detail to calculate the populations for the upper states of the Li-like satellites of the He- β with spectator electrons in $n = 2, 3$, and 4. For the plasma conditions of interest, the relative distribution of population within the $1s2\ell3\ell'$ or the $1s3\ell3\ell'$ doubly-excited states is close to LTE, while the populations of these two manifolds are not in LTE with respect to each other, nor are they in LTE with the upper state of the He- β . It is therefore important to do the non-LTE modeling to make a realistic estimate of the relative intensities of the satellite lines to their associated resonance lines.

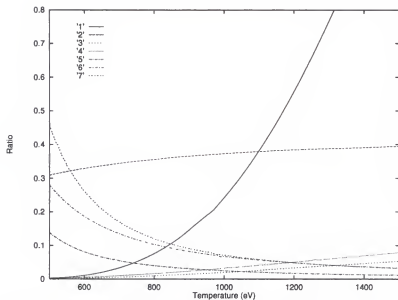


Figure 7. Intensity (relative to the Ar He- β intensity) vs. kT_e for 1-Ly- β , 2-He- γ , 3,4-He-like satellites of the Ly- β with spectator electron in $n = 3, 2$ respectively, 5-Li-like satellite of the He- γ with spectator electron in $n = 2, 6, 7$ -Li-like satellites of the He- β with spectator electron in $n = 3, 2$, respectively.

An important temperature dependence in our model is in the ratio of the Ly- β intensity to the He- β intensity since they correspond to different ioniza-

tion stages. At the temperatures of interest here, around 1 keV, the ionization potential of He-like Ar (4121 eV) produces a strong temperature dependence in the populations of the H- and He-like ionization stages which sharply constrains our temperature inferences. Also, the ratios of the intensities of the Li-like satellites to the He- β are temperature sensitive, and, for a given electron density, there is a narrow temperature range where both the intensity of the Ly- β and these satellites are comparable, thus the simultaneous observation of these features is a useful temperature diagnostic in itself. This is illustrated in Figure 7.

The non-LTE kinetics calculations were performed in the steady-state, optically thin approximation, as the spectra to be analyzed originate from plasmas with low concentrations of Ar, for which the ion motion effects on the lineshapes will be most noticeable. While at first glance these optically thin calculations seem reasonable, given the small fraction of Ar in D₂ for the plasmas of interest, they lead to a substantial underestimation of the intensity of the Lyman- β . The transfer of the optically thick ($\tau_0 \sim 10$) He- α line from the core leads to a shift in the ionization balance towards higher ionization stages which results in a diminution of He-like Ar, and an increase in H-like Ar. This effect on the resonance lines was estimated using an escape factor approximation³⁴. A comparison of the radiative transfer effect on line intensities as calculated with this escape factor method and by solving the linearized transfer equation is presented by Keane, *et al.*³⁵ To this end, escape factors for the He- α , He- β , He- γ and He- δ line transitions in He-like Ar, and Ly- α , Ly- β , and Ly- γ in H-like Ar were calculated using Stark broadened line profiles for several electron densities, n_e , assuming a uniform and spherical plasma, following the method described in Ref. 34. As an illustration of these results, escape factors for the

He- β line are displayed in Figure 8 as a function of $X = \frac{\tau_{\omega}}{\phi(\omega)} = \frac{\tau_o}{\phi_o} = \frac{\pi e^2}{mc} f N_l R$ (a variable proportional to the areal density of the lower level of the transition), where R is the radius of the plasma sphere.

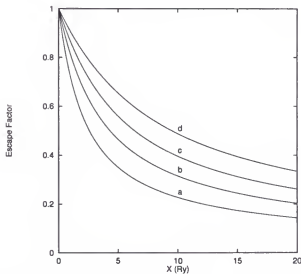


Figure 8. Escape factors for the He- β line vs. $X = \frac{\tau_{\omega}}{\phi(\omega)} = \frac{\tau_o}{\phi_o}$ for several values of the electron density n_e : (a) $2 \times 10^{23} \text{cm}^{-3}$ ($\phi_o = 0.2454 \text{Ry}^{-1}$), (b) $5 \times 10^{23} \text{cm}^{-3}$ ($\phi_o = 0.1392 \text{Ry}^{-1}$), (c) $1 \times 10^{24} \text{cm}^{-3}$ ($\phi_o = 0.1041 \text{Ry}^{-1}$), and (d) $2 \times 10^{24} \text{cm}^{-3}$ ($\phi_o = 0.0826 \text{Ry}^{-1}$).

The use of Stark broadened line profiles results in escape factors which are electron density dependent through the lineshapes (besides the dependence through τ_o): as n_e increases, changes in the line profiles (larger width, stronger wings) result in an increase in the escape factor values. These escape factors were used in the atomic kinetics calculations to account for opacity effects; they affect the computation of excited level populations as well as the total emergent line intensity.

The inclusion of this effect leads to model spectra which can fit simultaneously the relative intensities of the resonance lines as well as the relative

intensities of the resonance lines to their attendant satellites. Thus, while opacity has a small impact on the individual lineshapes, it is important in determining the relative intensities. The effect of radiative transfer on the resonance line intensities in our model is shown in Table 2, for two different Ar concentrations. This table lists C , the correction to the ratio of intensities due to radiative transfer, defined for line x as

$$C_x = \frac{\left(\frac{I_x}{I_{\text{Ly}-\beta}}\right)_{\text{thick}}}{\left(\frac{I_x}{I_{\text{Ly}-\beta}}\right)_{\text{thin}}},$$

where I_x is the integrated intensity of line x , and ‘thick’ and ‘thin’ denote the results of the optically thick and thin non-LTE model, respectively.

Table 2. Radiative transfer corrections to the ratios of the integrated intensities of the He-like resonance lines in the model to the integrated intensity of the Ar Ly- β , for $n_e = 8 \times 10^{23} \text{cm}^{-3}$, and different number concentrations of Ar in D₂ and temperatures.

kT_e	$C_{\text{He}-\beta}$		$C_{\text{He}-\gamma}$		$C_{\text{He}-\delta}$	
	0.25% Ar	1% Ar	0.25% Ar	1% Ar	0.25% Ar	1% Ar
500	0.76	0.42	0.89	0.67	0.91	0.73
600	0.59	0.29	0.71	0.48	0.74	0.53
700	0.50	0.27	0.60	0.43	0.62	0.48
800	0.48	0.29	0.56	0.44	0.57	0.48
900	0.49	0.34	0.55	0.48	0.56	0.51
1000	0.51	0.40	0.56	0.52	0.57	0.55
1100	0.54	0.45	0.58	0.56	0.58	0.58
1200	0.56	0.50	0.59	0.59	0.60	0.61

The sensitivity of the model spectrum to changes in density, and temperature is indicated in Figures 9 and 10. The positions of the centers-of-gravity of

the Ar resonance lines are indicated in Figure 9 by the locations of the centers of the labels.

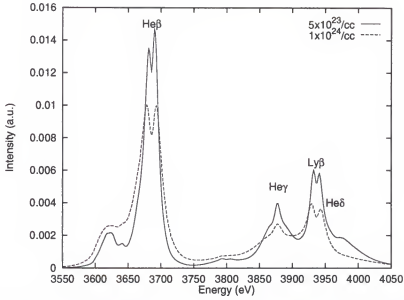


Figure 9. Density sensitivity of the model spectrum, $kT = 900\text{eV}$, .25% Ar in D_2 . The centers of gravity of the Ar resonance lines included in the model are indicated.

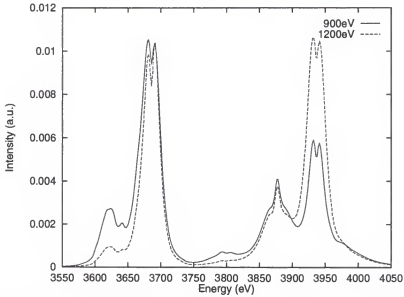


Figure 10. Temperature sensitivity of the model spectrum, $n_e = 5 \times 10^{23}\text{cm}^{-3}$, 1% Ar in D_2 .

Included in the model are five satellites to these resonance lines: the Li-like satellites of the Ar He- β with spectator electron in $n=2$ (c-o-g 3623eV) and $n=3$ (3669eV), the He-like satellites of the Ar Ly- β with spectator electron in $n=2$ (3871eV) and $n=3$ (3918eV), and the Li-like satellite of the Ar He- γ with spectator electron in $n=2$ (3797eV). The model spectrum shows sufficient temperature and density sensitivity to be useful in the analysis of plasmas whose electron temperatures range from 600 to 1200eV, and whose electron densities lie between 5 and $20 \times 10^{23} \text{cm}^{-3}$

Experimental Data

A series of laser-driven implosions was conducted at the Laboratory for Laser Energetics (LLE) at the University of Rochester in October and November of 1992. The series was designed, in part, to explore the inter-relationship between the contributions to spectral line broadening from dynamic ions and that from opacity. The targets were 250 μm diameter plastic microballons filled with 20atm of Ar and D₂ in varying concentrations, ranging from 0.025% Ar to pure Ar. The wide range of Ar concentrations was chosen so that the transition from ion-dynamic-dominated peaks to opacity dominated peaks could be studied. Core conditions at stagnation were such that K-shell Ar line emission was easily observable. Time resolved spectra were recorded using a flat-crystal spectrograph (FCS). The FCS has an ADP crystal with a 2-d spacing of 10.642 $\text{\AA}$ coupled to an x-ray streak detector. Instrument response is detector limited, *i.e.* streak camera resolution. With a target-to-detector distance of 300mm, the source size broadening is negligible.

This section discusses the data from 5 shots, taken from the high resolution FCS data. In all 5 shots, the FCS was imaging the spectral region from around

3400eV to 4200eV, where the Ar He- β , - γ , and - δ , and the Ly- β , along with their attendant satellites, are located. The instrumental response function is approximated by a Gaussian, whose full width at half maximum is energy dependent, ranging from 3.6eV at 3680eV to 4.4eV at 3935eV.

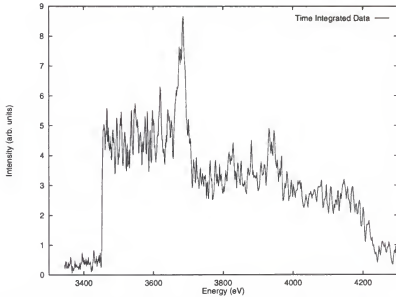


Figure 11. Time-integrated spectral data from shot 24544. The fill gas was 20 Atm D₂ doped with 0.01 Atm Ar. The shell thickness was 6 microns, and the total blue energy on target was 809.5 J.

The lowest concentration of Ar in D₂ in this series of experiments was 0.01atm of Ar in 20atm of D₂, a number concentration of 0.025% Ar. In this shot, the attempt to observe diagnostically useful Ar line emission was unsuccessful (see Figure 11).

With ten times as much Ar as in the previous target, the implosion with 0.25% Ar in D₂ (designated A) yielded the useful time dependent spectra displayed in Figure 12. Each of the 5 lineouts shown represent an average over approximately 50ps, and each overlaps by about 25ps with neighboring lineouts.

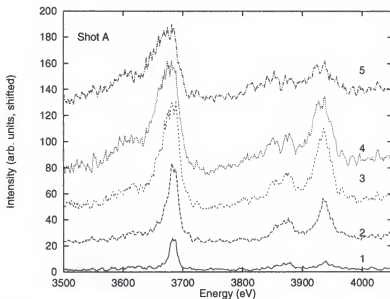


Figure 12. Time resolved lineouts from Shot A (shot number 24545, 0.25% Ar in D₂, 6 micron shell, 782.4 J blue energy on target). Labels used in text to refer to individual lineouts. Individual lineouts are shifted vertically by an arbitrary amount for display purposes. The five lineouts shown each average over approximately 50ps, and the time intervals over which they average overlap by 25ps with neighboring lineouts.

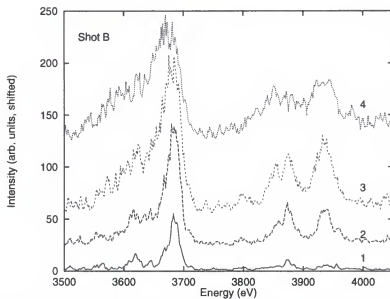


Figure 13. Time resolved lineouts from Shot B (shot number 24537, 1% Ar in D₂, 6 micron shell, 862.3 J blue energy on target). Labels used in text to refer to individual lineouts. Individual lineouts are shifted vertically by an arbitrary amount for display purposes. The four lineouts shown each average over approximately 25ps, with no overlap between consecutive time intervals.

The two shots with 0.4atm of Ar in 19.6atm D₂ (a concentration of 1% Ar in D₂) produced enough signal such that the window over which the time integration was done could be reduced to 25ps, with no overlap. Figure 13 shows the time resolved spectra from one of these shots, designated B.

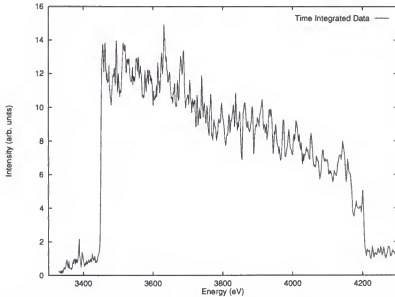


Figure 14. Time-integrated data from shot 24543 (5.0 Atm Ar + 15 Atm D₂, 6 micron shell, 775.9 J blue energy on target).

With 5atm of Ar in D₂ (17% Ar in D₂), the last implosion considered here did not achieve high enough temperatures to produce useful Ar emission spectra in our range of interest possibly due to a combination of the increased target mass and radiative cooling.

Analysis

The model spectrum is now used to analyze the data from the shots designated A and B, concentrating on two aspects of the analysis. First, the composite spectral feature formed by the He- β line and its Li-like satellites is

examined for the effects of opacity and ion dynamics. The temperature- and density-sensitivity of this composite feature has been published previously, in the static-ion optically thin approximation^{36,37}. This composite spectral feature, the most prominent feature in all the spectra we consider, best illustrates the importance of both ion dynamics and opacity broadening on the lineshape. Secondly, the utility of the model spectrum is illustrated by attempting to fit the spectral region from 3500eV to 4100eV for a lineout from shot B. Preliminary results from the use of the model spectrum, absent the effects of radiative transfer on the intensity ratios, have been presented previously³⁸.

Before comparison, the model spectrum was convolved with our approximation to the instrumental response function, and a background was subtracted from the experimental spectrum. The model spectrum was shifted arbitrarily by between 1 and 5 eV in the direction of lower energy to match the locations of the lines in the experimental data. This arbitrary shift of the model spectrum is now understood to represent an approximate accounting for the shifts due to the radiators interaction with the plasma electrons³⁹

The role that ion dynamics plays in the formation of the line emission spectra for low concentrations of Ar in D₂ is indicated in Figure 15, where the He- β + Li-like satellites composite spectrum from lineout 3 of shot A (0.25% Ar in D₂) is compared to our model calculation. In Figure 15, the effect of opacity on the lineshape is seen to be minimal, as the small concentration of Ar prevents significant redistribution of intensity. However, for this low concentration of Ar in D₂, opacity alone can not account for the lack of a central dip in the observed He- β . When the effect of ion dynamics is included, the dip in the model spectrum is commensurate with the experimental data. Analysis of other lineouts from this shot also indicate that opacity alone can

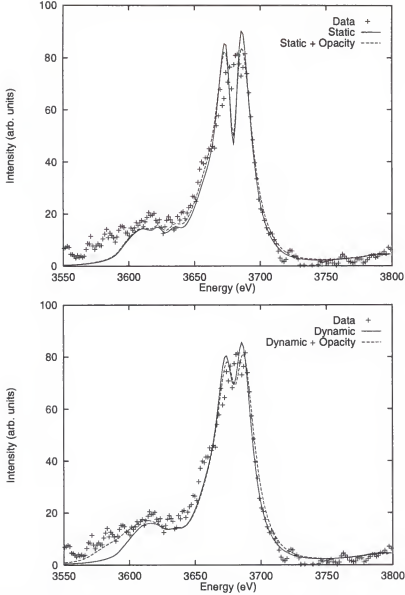


Figure 15. Detail from lineout A-3 (0.25% Ar in D₂) and fits using differing approximations to calculate model spectrum, $n_e = 8 \times 10^{23} \text{cm}^{-3}$, $kT = 925 \text{eV}$. Only the fit labeled “Dynamic + Opacity” includes the correction for satellite emission in the 3550-3600eV region, as discussed in the text.

not explain the lack of a large central dip in the He- β line, while the inclusion of ion dynamic effects in the line shape calculation leads to spectra which compare well with the experimental data.

The first three fits in Figure 15 (Static, Static + Opacity, and Dynamic) indicate a deficiency in the model spectrum. The model predicts negligible emission in the region from 3550eV to 3600eV, on the red side of $n=2$ Li-like satellite of the $\text{He-}\beta$. As the Li-like satellites obviously contribute to the emission on the red side of the $\text{He}\beta$, it is natural to consider the importance of Be-like satellites in the spectral region where the model is deficient. There are transitions in this region in Be-like Ar from the $1s2\ell2\ell'3\ell''$ manifold to the $1s^22\ell2\ell'$ manifold. Though the kinetic model did not include sufficient detail in the Be-like ionization stage to calculate the intensity of this line, we can include it in the model spectrum with its intensity a free parameter. Such an inclusion is justified by data from similar shots where the FCS was imaging the α lines of Ar. In those shots, the emission from the Be-like satellite of the $\text{He-}\alpha$ is apparent. The final fit in Figure 15 (Dynamic + Opacity) includes an estimate of the intensity from the Be-like satellite, as a preliminary attempt to assess the importance of emission from this satellite.

Shot B, with four times the Ar concentration of shot A, allows the exploration of the importance of ion dynamics on the lineshape when opacity is not negligible. Figure 16 is similar to Figure 15, in that it compares the model spectra to the $\text{He-}\beta$ + Li-like satellites composite spectral feature from the data. As the concentration of Ar in D_2 increases, the effect of opacity on the lineshape becomes more important, and ion dynamics has less of an effect on the peak structure. The density inference from this lineout depends on the inclusion of opacity, as the effect of opacity, while most noticeable on the peak structure, is also important for the overall line shape.

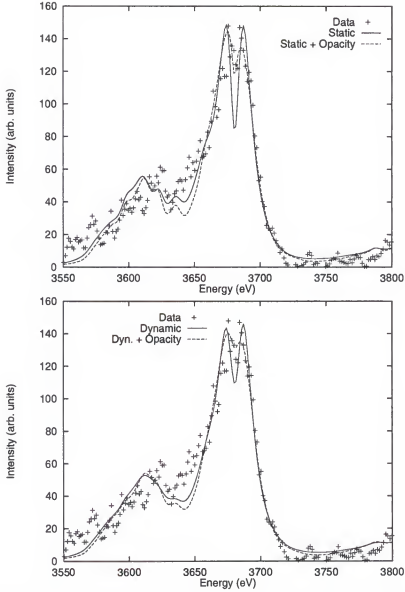


Figure 16. Detail from lineout B-3 (1% Ar in D_2), and fits using model spectrum. Static: $n_e = 8 \times 10^{23} \text{cm}^{-3}$, $kT = 875 \text{eV}$; Static + Opacity: $n_e = 5 \times 10^{23} \text{cm}^{-3}$, $kT = 840 \text{eV}$; Dynamic: $n_e = 8 \times 10^{23} \text{cm}^{-3}$, $kT = 875 \text{eV}$; Dynamic + Opacity: $n_e = 5 \times 10^{23} \text{cm}^{-3}$, $kT = 840 \text{eV}$.

In Figure 16, there is a noticeable discrepancy between the calculated spectrum and the experimental data, in the spectral region around 3640 eV. There is emission in this region in the data which is not present in our model. While

this is the most blatant occurrence of this discrepancy, other lineouts also display greater intensity in this region than the present calculations predict. The unaccounted for emission occurs in the location of the blue peak of the $n=2$ Li-like satellite of the He- β . The discrepancy is greatest for low density conditions, occurring early in the implosions. We conjecture that non-LTE or time dependent effects or the unresolved $n = 4$ Li-like satellite of the He- β may account for the extra emission in this spectral region.

Having explored the importance of ion dynamics and opacity on the line-shape, I turn to an examination of the utility of our model spectra as a tool for the inference of plasma temperature and density. The present calculation models line emission from the spectral region 3550eV to 4000eV, including the Ar Ly- β line and its He-like satellites with spectator electron in $n=2$ and 3, the Ar He- β and its Li-like satellites with spectator in $n=2$ and 3, the He- γ line and its $n=2$ Li-like satellite, and the He- δ . Be-like emission around 3580eV is accounted for in the manner indicated before.

The density inference is most easily made by fitting the blue wings of the He- β and the Ly- β . These wings are less effected by the presence of satellites than the red wings, though care must be taken to include the He- δ line when considering the blue side of the Ly- β . These wings display enough density sensitivity to distinguish between lines emitted from plasmas whose densities differ by a factor of 1.5 in this regime of plasma conditions. Thus, this technique can distinguish between emission from lineouts characteristic of 5, 8 and $10 \times 10^{23} \text{cm}^{-3}$.

Given a density inference, we can inquire about the average temperature of the emitting region. In Figure 17, we fit the entire spectral region from 3550eV to 4000eV for the third lineout from shot B.

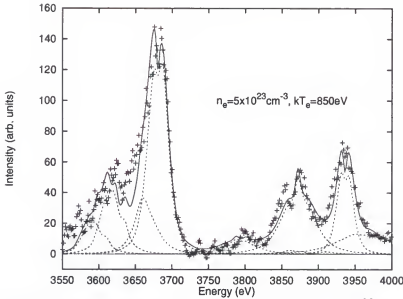


Figure 17. Best fit to lineout B-3 (1% Ar in D₂), $n_e = 5 \times 10^{23} \text{cm}^{-3}$, $kT = 850 \text{eV}$. The dashed traces are the individual lineprofiles that were added to produce the model spectrum.

The temperature inference from this fit is within 10% of those obtained by separately fitting the region containing only the He- β and its Li-like satellites, and the region containing only the Ly- β , the He- γ and - δ , and their satellites. This degree of consistency is typical of the results for this type of test. This accuracy in temperature inference can be attributed to the strong constraints put on the fit, in that it has to reproduce emission emanating from transitions in three different ionization stages in Ar. Indeed, it was just this constraint which led to the current appreciation of the importance of the radiative transfer effect on the intensity of the Ly- β relative to the intensities of the other lines in the model. Without the inclusion of this effect, it proved impossible to fit the intensity of the Li-like satellites of the He- β and the Ly- β simultaneously with a single temperature.³⁸ The density and temperature inferences obtained from our analysis of shots A and B are presented in Table 3.

Table 3. Inferences of plasma conditions obtained by comparing model spectra with the experimental data of Figures 9 and 10, over the spectral range 3550eV to 4000eV.

Shot A 0.25% Ar in D ₂			
Lineout	Density (10^{23}cm^{-3})	Temperature (eV)	Diameter (μm)
1	< 5 a		
2	5	1000	32
3	8	1050	27
4	10	1025	25
5	10 b	950	25
Shot B 1% Ar in D ₂			
Lineout	Density (10^{23}cm^{-3})	Temperature (eV)	Diameter (μm)
1	$\ll 5$ a		
2	< 5 c		
3	5	850	33
4	8	880	28

- a The early lineouts come from times when the electron density was less than $5 \times 10^{23}\text{cm}^{-3}$, the lowest electron density we considered when preparing our model spectrum.
- b The fit at $10 \times 10^{23}\text{cm}^{-3}$ indicates that the electron density is slightly higher.
- c Although this lineout is remarkably well fit with a optically thin model spectrum with an electron density of $5 \times 10^{23}\text{cm}^{-3}$, and electron temperature of 800eV, the amount of opacity which we calculate for the lines at these conditions is appreciable, and leads to lines which are overly broad.

Comments

From the preceding analysis, it is clear that a model combining the intertwined effects of ion dynamics and opacity is required for consistent fitting of the spectral data from implosions where a small amount of Ar is added to the fuel for diagnostic purposes. For the 0.25% Ar case, the effect of ion dynamics

on the lineshape was necessary to model the peaks in the experimental data. If static lineshapes are used to fit the data for this case, one would infer optical depths and thus plasma diameters which are too large by a factor of two to be consistent with the diameters measured by x-ray microscopes.³⁸ For the 1% Ar case, the effect of opacity on the lineshape was important to fit the peaks in the data, though the effects of ion dynamics were not negligible. Also, in both cases the radiative transfer effect on the relative intensities of the lines is important in providing consistent relative intensities for the resonance lines and their satellites.

The results presented here do illustrate the importance of ion dynamics on the Ar lineshapes emitted from deuterium plasmas doped with trace amounts of Argon. However, the resolution of the experimental data near the peaks of the resonance lines is not sufficient to distinguish between differing theories of ion dynamics. Higher resolution spectroscopy of, say, the He- β would go far in testing the detailed effect of ion dynamics on the lineshape, and the results presented here point to plasma compositions and conditions where such experiments could be carried out fruitfully. Another advantage of higher resolution spectroscopy would be the ability to probe the effects of the real part of $M(\Delta\omega)$, both on the location and shapes of the lines.[†]

[†] Only the imaginary part of $M(\Delta\omega)$, that part leading to electron broadening, is included in our calculation. Although shifts due to the radiator-plasma electron interaction have received considerable theoretical attention (see, *e.g.*, H.R. Griem *et al.*, Phys. Rev A **41**, 5600 (1990), M. Koenig *et al.*, Phys. Rev. A **38**, 2089 (1988), J. Cooper, in *Radiative Properties of Hot Dense Matter* eds. J. Davis, *et al.*, World Scientific (Singapore, 1985)), we found no compelling experimental evidence here of overall shifts due to this interaction, nor of additional modification of the lineshape due to differential shifting of individual components. However, recent analysis of indirect drive experiments designed to reduce the effects of gradients show both shifts and distortions, particularly of the He- γ line.³⁹

Currently, a detailed comparison of the results of the method used here to include ion dynamic effects in the calculation of Stark broadened lineshapes with results of a different method⁴⁰ is underway. This different method is the frequency fluctuation model, in which the effect of the slow modulation of the ion microfield is approximated as a stochastic process mixing the components of the static lineshape.

Mention should be made of the efforts of a group at LLNL to address the question of ion dynamics⁴¹. Rather than change the relative perturber/radiator concentration to modify the effects of ion dynamics, at the cost of changing the opacity, as was done in this study, the LLNL group kept the amount of radiator constant, and changed the perturber species, thus changing the mass and the diffusion coefficient while leaving the Ar opacity unchanged. Unfortunately, this change in perturber species led to hydrodynamic differences between shots which made the assessment of ion dynamic effects difficult. They find that there is no “dip” in the He- β line even for those cases where the perturber mass is such that they expect minimal ion dynamic effects. They attribute this to gradients in the plasma, and criticize the work presented here for not accounting for gradients. I find no compelling evidence that the spectra indicate the presence of gradients, and am satisfied that the uniform core assumption, along with careful treatment of opacity and ion dynamic effects, explain the main features of the spectrum. This point is discussed again in the final chapter of this dissertation.

The model spectrum is calculated assuming that the radiating region of the plasma can be characterized by a single temperature and density. While the model spectrum produced using this assumption is capable of reproducing

many of the features of the experimental data, some aspects of the fits, particularly in the region around the peak of the He- γ are unsatisfactory. Work is in progress to assess the importance of gradients. Preliminary studies modeling radiation from a layered core structure appear promising. Also, the real part of the radiator-plasma electron interaction, that part responsible for the shifting of the individual components of the line, has recently been calculated for the Ar He- γ line³⁹, and the results are such that the fit to experimental data similar to that presented here is much improved by the inclusion of this effect.

CHAPTER 4 ABSORPTION SPECTROSCOPY

Absorption spectroscopy has become an important tool in the analysis of laser-produced plasmas. In the past, absorption spectroscopy has been used to diagnose regions in the pusher of laser-compressed microballoons where self-emission is not useful for diagnostic purposes.^{42,43} More recently, experiments using layered planar targets have been diagnosed by absorption spectroscopy to determine temperatures in the radiatively heated zone located in the overdense plasma.⁴⁴ Analysis of these experiments relied upon detailed atomic physics calculations to model complex absorption spectra. In turn, these models were used to make quantitative estimates of the temperature distribution and ionization balance in the absorbing medium.^{45,46,47,48} Also, experiments using x-rays to heat the absorbing medium have been performed in connection with opacity measurements.^{49,50,51}

In all the aforementioned work, the absorbing media were not dense enough for Stark broadening to be important. The widths of the absorption features were density independent and due primarily to Doppler and instrumental effects. Thus, analyses of the absorption spectra could be performed, yielding information about the temperature conditions in the absorbing region, while information regarding the density of the absorbing media had to come from other considerations.

During the analysis of x-ray spectra obtained from implosion experiments conducted at the Laboratory for Laser Energetics at the University of Rochester,

features interpreted as absorption into auto-ionizing states were observed.⁵² A model for analysis of absorption features was developed and applied to these spectra. In order to reproduce the experimental data, the model included absorption of the radiation emitted by the core through a layered structure, each layer characterized by an electron density and temperature, and a geometrical length. As the number of layers increased, the model was able to produce increasingly accurate fits. The question remained as to whether the inferences from a multilayer model, with all its free parameters, could be relied upon. This chapter carefully develops and examines this model, applies it to synthetic spectra for which the layer structures are known, and draws conclusions as to the utility of layered models for the analysis of absorption spectra.

Stark Broadened Absorption Line Profiles

This development begins by relating the photoexcitation cross-section to the lineshape formula discussed in previous chapters. The photoexcitation cross-section for an ion in state i to absorb a photon of energy $h\nu$, ending in a state j , $\sigma_{ij}(\nu)$, is given by

$$\sigma_{ij}(\nu) = \frac{\pi e^2}{mc} f_{ij} \delta(\nu - \nu_{ij})$$

where $h\nu_{ij}$ is the energy of the transition, f_{ij} is the absorption oscillator strength,

$$f_{ij} = \frac{2m}{3\hbar^2 e^2} h\nu_{ij} | \langle i | \vec{d} | j \rangle |^2,$$

and e and m are the magnitude of the electron charge and the electron mass, respectively, c is the speed of light and $\vec{d}$ is the dipole moment of the absorbing ion. If this expression is multiplied by the density matrix element associated with the probability that the ion is in state i , ρ_i , the resulting expression is the

effective photoionization cross section for that ion and that transition. Summing this over all transitions that lie in a given frequency range, the frequency dependent photoexcitation cross-section for the ion, $\sigma_T(\nu)$, is

$$\sigma_T(\nu) = \frac{\pi e^2}{mc} \frac{2m}{3\hbar^2 e^2} \sum_{i,j} \rho_i \hbar \nu_{ij} \delta(\nu - \nu_{ij}) |< i|\vec{d}|j>|^2.$$

To render this expression in a form amenable to Stark broadening calculations, note that the primary frequency dependence of this cross section is due to the delta functions, provided that the frequency range is sufficiently narrow. In that case, one can approximate the cross-section as follows:

$$\sigma_T(\nu) \cong \frac{\pi e^2}{mc} \frac{2m}{3\hbar^2 e^2} \hbar \nu \sum_{i,j} \rho_i \delta(\nu - \nu_{ij}) |< i|\vec{d}|j>|^2.$$

To relate this to the lineshape, $\varphi(\nu)$, defined as

$$\varphi(\nu) = \sum_{i,j} \rho_i \delta(\nu - \nu_{ij}) |< i|\vec{d}|j>|^2,$$

note that this expression is used for both emission and absorption line profiles, the only difference being which set of states is being averaged over (initial states) and which is summed over (final states). Using this definition of $\varphi(\nu)$, write $\sigma_T(\nu)$ as

$$\sigma_T(\nu) = \frac{\pi e^2}{mc} \frac{2m}{3\hbar^2 e^2} \hbar \nu \varphi(\nu).$$

Multiplying and dividing the r.h.s of this equation by $\int \varphi(\nu) d\nu$ and denoting the area normalized line profile as

$$\hat{\varphi}(\nu) = \frac{\varphi(\nu)}{\int \varphi(\nu) d\nu}$$

one obtains

$$\sigma_T(\nu) = \frac{\pi e^2}{mc} \hat{\varphi}(\nu) \frac{2m}{3\hbar^2 e^2} \hbar \nu \int d\nu \sum_{i,j} \rho_i \delta(\nu - \nu_{ij}) |< i|\vec{d}|j>|^2.$$

Again using the approximation that $h\nu$ varies negligibly in the ν range of interest, one can bring the $h\nu$ inside the integral, perform the integration, and write (recalling the definition of f_{ij}):

$$\sigma_T(\nu) = \frac{\pi e^2}{mc} \hat{\varphi}(\nu) \sum_{i,j} \rho_i f_{ij}.$$

Finally, define an average or effective oscillator strength for the set of transitions from the set of initial states to the set of final states, $\bar{f}$, as

$$\bar{f} = \sum_{i,j} \rho_i f_{ij}$$

and write $\sigma_T(\nu)$ as

$$\sigma_T(\nu) = \frac{\pi e^2}{mc} \bar{f} \hat{\varphi}(\nu).$$

For the specific diagnostic application that is the focus of this chapter, the plasma conditions will be such that one may calculate the lineshape in the static ion approximation. The effects of autoionizing transitions on the lineshape will be approximated using the convolution method. The line shapes were thus calculated using a multi-electron radiator line profile formalism and code (MERL).^{13,12} Specifically, absorption line profiles for inner shell $n = 1$ to $n = 2$ transitions in the L-shell ions of Ar are calculated. The temperature, kT , range of interest was 100-600eV, and the electron number density, n_e , ranged from $5 \times 10^{23} \text{cm}^{-3}$ to $5 \times 10^{24} \text{cm}^{-3}$. Doppler broadening is included by convolving the Stark profile with a Gaussian of full width at half maximum (FWHM) between 0.5 and 1.2 eV, appropriate for Ar ions with kT in the 100-600eV range and photon energies of $h\nu \approx 3100 \text{eV}$. Ion broadening is treated in the static ion approximation and the effects of dynamic perturbing electrons are included using a second order relaxation theory. Ion- and electron-radiator interactions are computed in the dipole approximation. Necessary atomic physics data

(energy level structure, dipole and quadrupole reduced matrix elements) were calculated using Cowan's multiconfiguration atomic structure codes including relativistic corrections.⁵³

As an illustration of the results, Figure 18 shows absorption line profiles for Be-like Ar at a temperature of 400eV and for electron densities of 0.5, 1, 2, and $5 \times 10^{24} \text{cm}^{-3}$.

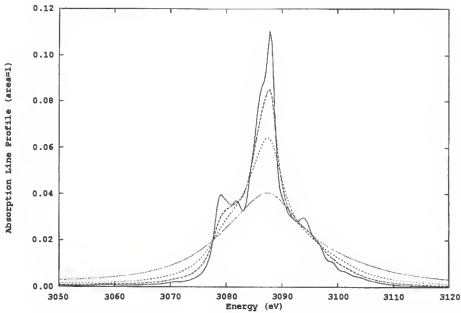


Figure 18: Area-normalized absorption line profiles for Be-like Ar for $kT = 400\text{eV}$ and $n_e = 0.5 \times$ (solid), $1.0 \times$ (long dashes), $2.0 \times$ (short dashes), and $5.0 \times 10^{24} \text{cm}^{-3}$ (dots).

These calculations include all possible configurations with only $n = 1$ and $n = 2$ electrons: the initial state configurations $1s^2 2s^2$, $1s^2 2s 2p$, and $1s^2 2p^2$ (10 lower energy levels), final state configurations $1s 2s^2 2p$, $1s 2s 2p^2$, and $1s 2p^3$ (30 upper energy levels), and a total of 83 transitions. Figure 18 clearly displays the

electron density sensitivity characteristic of the line profiles we consider here: as n_e increases, the line profile lose the jaggedness characteristic of low densities, becoming broader and smoother. The weak kT dependence of the line profile is illustrated in Figure 19, where Be-like absorption lines at $n_e = 1 \times 10^{24} \text{cm}^{-3}$ and $kT = 100, 200, 400$, and 600 eV are shown.

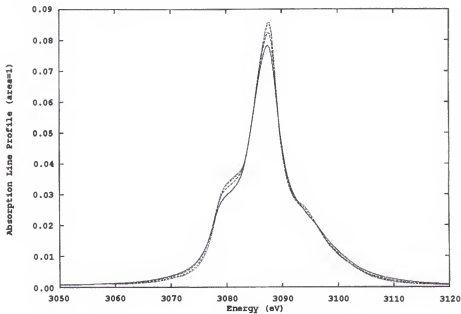


Figure 19: Area-normalized absorption line profiles for Be-like Ar for $n_e = 1.0 \times 10^{24} \text{cm}^{-3}$ and $kT = 100$ (solid), 200 (long dashes), 400 (short dashes), and 600 eV (dots).

Another example is shown in Figure 20: C-like Ar. In this case, the initial state configurations are $1s^2 2s^2 2p^2$, $1s^2 2s 2p^3$ and $1s^2 2p^4$ (20 lower energy levels), the final state configurations are $1s 2s^2 2p^3$, $1s 2s 2p^4$, and $1s 2p^5$ (30 upper energy levels), there are a total of 150 transitions. This figure shows the same characteristic behavior: strong electron density sensitivity.

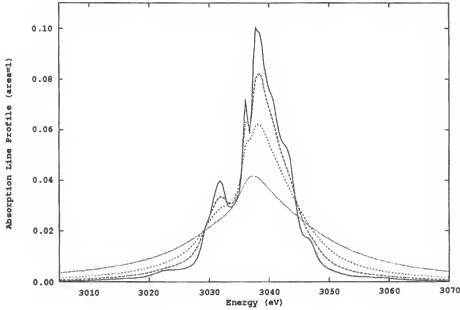


Figure 20: Area-normalized absorption line profiles for C-like Ar for $kT = 300$ eV and $n_e = 0.5 \times$ (solid), $1.0 \times$ (long dashes), $2.0 \times$ (short dashes), and $5.0 \times 10^{24} \text{cm}^{-3}$ (dots).

In these line profiles, broadening effects were considered in both the initial and final states of the transitions. For the plasma conditions considered here, the dynamic electrons are primarily responsible for the broadening of the line profiles because the ion microfield distribution broadens the line profile only slightly and shifts it toward higher energies by a small amount. These effects are illustrated in Figures 21 and 22.

To check the effect of the ion-quadrupole-field gradient interaction in the line profiles the Li-, Be-, B-, and C-like Ar cases were recomputed including this interaction at electron densities of $1 \times 10^{24} \text{cm}^{-3}$ and $5 \times 10^{24} \text{cm}^{-3}$; the field gradients were estimated with the aid of an improved model that includes

ion correlation effects⁵⁴. In all cases, it was found that the ion quadrupole interaction produces only small changes in these lineshapes.

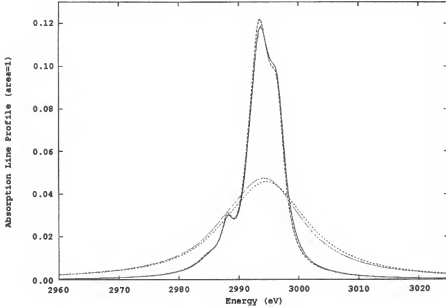


Figure 21: Area-normalized absorption line profiles for O-like Ar for $kT = 300\text{eV}$. $n_e = 1 \times 10^{24}\text{cm}^{-3}$ with ion microfield integration (solid) and without (short dashes). $n_e = 5 \times 10^{24}\text{cm}^{-3}$ with ion microfield integration (long dashes) and without (dots).

The broadening of the lineshape due to the Auger process and the radiative decay of the upper energy levels is negligible compared to the broadening due to the Stark and Doppler effects for the temperatures and densities considered here. The results of calculations of B-like Ar absorption features including and ignoring the broadening due to the Auger and radiative widths are shown in Figure 23 for the lowest density considered here, $5 \times 10^{23}\text{cm}^{-3}$. The Auger and radiative widths were taken from the work of Chen and Crasemann⁵⁵.

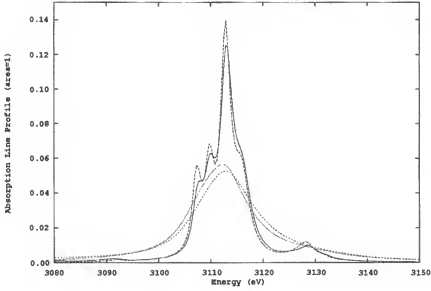


Figure 22: Area-normalized absorption line profiles for Li-like Ar for $kT = 400\text{eV}$. $n_e = 1 \times 10^{24}\text{cm}^{-3}$ with ion microfield integration (solid) and without (short dashes). $n_e = 5 \times 10^{24}\text{cm}^{-3}$ with ion microfield integration (long dashes) and without (dots).

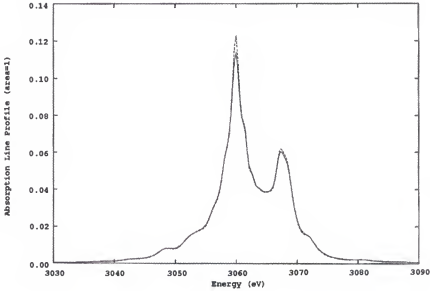


Figure 23: Area-normalized absorption line profiles for B-like Ar for $kT = 400\text{eV}$ and $n_e = 5 \times 10^{24}\text{cm}^{-3}$ with (solid) and without (dashes) the effects of the Auger and radiative widths included.

Optical Depth of a Uniform Absorbing Region

The Stark broadened absorption line profiles for the L-shell ions of Ar were used to calculate the frequency-dependent optical depth of a uniform absorbing layer of a given electron density, n_e , temperature, kT , and thickness, l . Using the results of the previous section, the optical depth due to absorption by ions in a particular ionization stage, *e.g.*, Be-like Ar, $\tau_{\text{Be}}(\nu)$, is given by

$$\begin{aligned}\tau_{\text{Be}}(\nu) &= \tau_{\text{Be}}(\nu; n_e, kT, l) \\ &= \sigma_{T, \text{Be}}(\nu; n_e, kT, l) N_{i, \text{Be}} l \\ &= \frac{\pi e^2}{mc} f_{\text{Be}}^- \hat{\phi}_{\text{Be}}(\nu; n_e) N_{i, \text{Be}} l,\end{aligned}$$

where $N_{i, \text{Be}}$ is the number density of ions in the initial states of the inner-shell transitions in Be-like Ar and where the temperature and density dependences of the lineshape and the distribution of populations are explicitly displayed. The following discussion ignores the weak temperature dependence of the lineshape for the plasma conditions considered here.

For cases in which absorption due to transitions in two or more ionization stages is important, the optical depths due to all relevant ionization stages are added to yield the total optical depth of the layer. Hence, for absorption due to transitions in the L-shell ions of Ar,

$$\tau(\nu) = \tau_{\text{F}}(\nu) + \tau_{\text{O}}(\nu) + \cdots + \tau_{\text{Li}}(\nu) + \tau_{\text{He}}(\nu)$$

where F refers to Fluorine-like Ar, O to Oxygen-like Ar, *etc.* (While, taxonomically speaking, the He-like ionization stage is K-shell rather than L-shell, we include it here as there are temperatures and densities within our range of interest for which $1s^2 \rightarrow 1s2\ell$ absorption is non-negligible.) The total optical depth of the layer thus has contributions from all transitions in all the relevant

ionization stages at the given values of kT and n_e . Effective absorption oscillator strengths, $\bar{f}$, for these transitions have been calculated for all Ar L-shell ions, and the results are given in Table 4.

Table 4: Effective oscillator strengths for the $n = 1 \rightarrow 2$ transitions in Ar ions.

Argon Ion	$\bar{f}$
He-like	0.8041
Li-like	0.6923
Be-like	0.5754
B-like	0.4643
C-like	0.3591
N-like	0.2600
O-like	0.1671
F-like	0.0803

The distribution of population among the initial states of the absorption transitions is calculated with the Saha equation. For the temperatures and densities considered here, the results of this LTE approximation have been compared to the corresponding results from an NLTE model³³, and agreement, within 5%, was found between the predicted populations of the initial states of the transitions.

Using these assumptions, the total optical depth of a uniform absorbing layer due to $n = 1$ to $n = 2$ inner-shell transitions in the L-shell Ar ions can be calculated as a function of the temperature, density, and thickness of the layer. It is appropriate to emphasize that the optical depth incorporates the temperature and density dependence of the distribution of population (through

N_i) and the density dependence of the Stark broadening effect (through $\phi(\nu)$). Figure 24 displays the optical depth as a function of photon energy at two temperatures for fixed density and layer thickness, $n_e = 5 \times 10^{23} \text{cm}^{-3}$, $l = 1 \mu\text{m}$.

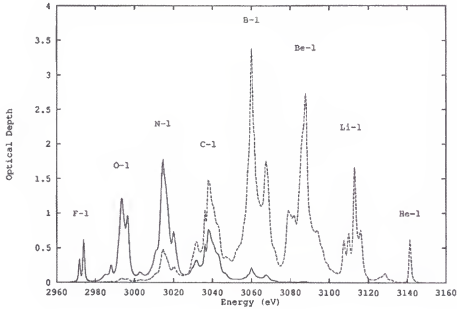


Figure 24: Optical depth, $\tau(\nu)$, for $n_e = 0.5 \times 10^{24} \text{cm}^{-3}$, $l = 1 \mu\text{m}$ and $kT = 150 \text{eV}$ (solid) and 225eV (dashes). The labels above the absorption features indicate the Ar ion that is the dominant absorber in that region of the spectrum.

The change in τ as the temperature rises reflects the shift in ionization balance, for $kT = 150 \text{eV}$ the absorption spectrum is dominated by F-, O-, N-, and C-like Ar, while for $kT = 225 \text{eV}$ C-, Be-, B-, and Li-like Ar are prominent. Note that, for the later case, the maximum values of $\tau(\nu)$ for absorption by the dominant

ions are in general larger than in the former case because of an increase in absorption oscillator strength.

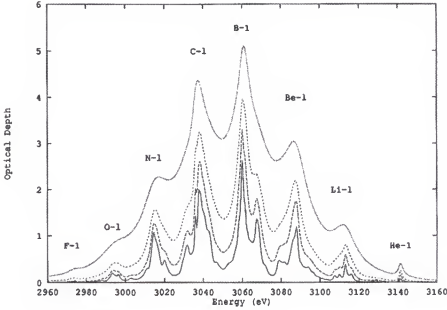


Figure 25: Optical depth, $\tau(\nu)$, for $l = 1\mu\text{m}$ and $0.5 \times 10^{24}\text{cm}^{-3}$, 200eV (solid), $1.0 \times 10^{24}\text{cm}^{-3}$, 235eV (long dashes), $2.0 \times 10^{24}\text{cm}^{-3}$, 278eV (short dashes), $5.0 \times 10^{24}\text{cm}^{-3}$, 353eV (dots).

More interesting from the point of view of Stark broadening is the dependence of the optical depth on electron density. Changing the electron density affects both the ionization balance and the Stark broadening of the absorption line profiles. For a given temperature, increasing the electron density will shift the ionization balance downward as the increase in three-body recombination will preferentially populate those ionization stages with more bound electrons. The increase in electron density will also increase the broadening of the absorption features. To focus on the broadening effect, Figure 25 shows the results

for a layer thickness of $1\mu\text{m}$ and several combinations of temperature and density chosen such that the ionization balance remains approximately constant; hence, in each case, the optical depth is dominated by the same ions.

As the density increase from $5 \times 10^{23}\text{cm}^{-3}$ to $5 \times 10^{24}\text{cm}^{-3}$, the absorption feature becomes less structured and the width of the absorption peaks becomes broader due to the density dependence of the lineshapes. Note that, due to the density dependence of the lineshapes, the values of the local maxima and minima do not increase linearly with density. For example, $\tau(\nu)$ at the peak of the absorption due to B-like Ar for the $5 \times 10^{23}\text{cm}^{-3}$ curve is 2.6, and when the density is increased by a factor of 10 this same peak has a $\tau(\nu)$ of 5.1. The peaks of the optical depth increase by factors of between 2 and 4. This reflects the competition between the two effects: the increase in the number of absorbing ions, which would tend to increase the optical depth, and the increase in the plasma perturbation on the energy levels of the transitions, which would tend to broaden the lineshape and lower the peak optical depth. These effects which compete at the peaks cooperate at the “dips”, where the optical depths increase by a factor between 12 and 20 when the density is increased ten-fold.

Analysis of Absorption Spectra

Using the above theory for the optical depth due to a single uniform layer, we can model attenuation through an absorbing layer with nonnegligible temperature and/or density gradients by stacking successive uniform layers, each characterized by a set of kT , n_e , and l , and allowing the radiation transmitted through the first layer to be further attenuated by the second layer, and so on. Thus, we have for the intensity, $I(\nu)$, transmitted by a stack of N layers

$$I(\nu) = I_0(\nu)e^{-\tau^1(\nu)}e^{-\tau^2(\nu)}\dots e^{-\tau^N(\nu)} = I_0(\nu)e^{-\tau^T(\nu)}$$

where $I_0(\nu)$ is the incident intensity,

$$\tau^i(\nu) = \tau^i(\nu; n_e^i, kT^i, l^i)$$

is the optical depth due to the i th layer, and $\tau^T(\nu) = \sum \tau^i(\nu)$ is the total model optical depth at frequency ν .

The temperature and density dependence of the optical depth can be exploited to analyze absorption spectra [*e.g.*, an experimental spectrum, $I^{\text{exp}}(\nu)$] to diagnose the environmental conditions within the absorbing plasma. If the experimental spectrum, $I^{\text{exp}}(\nu)$, is expressed

$$I^{\text{exp}}(\nu) = I_0^{\text{exp}}(\nu)e^{-\mathcal{T}^{\text{exp}}(\nu)}$$

we may approximate $\mathcal{T}^{\text{exp}}(\nu)$ using our modeled $\tau^T(\nu)$. To infer $\mathcal{T}^{\text{exp}}(\nu)$ from $I^{\text{exp}}(\nu)$, it is necessary to know $I_0^{\text{exp}}(\nu)$; in some cases, this knowledge can be acquired experimentally; otherwise, a spectral analysis package, such as ROBFIT⁵⁶ can be used. By examining the experimental spectrum, one can determine the ionization stages and transitions responsible for the absorption features. The width and strength of these features then allow for an estimate of the density and temperature ranges in the absorbing region, setting bounds on the parameters chosen to fit the spectrum. To approximate $\mathcal{T}^{\text{exp}}(\nu)$ using our model calculations, we choose a set of N layers with given temperatures and densities and minimize the sum of squares,

$$Q^2 = k \sum_{j=1}^M (\mathcal{T}^{\text{exp}}(\nu_j) - \tau^T(\nu_j))^2$$

(where k is a constant for a given experimental spectrum) with respect to the set of layer thicknesses. Because $\tau^T(\nu)$ depends linearly on the l^i s, this Q^2 minimization can be performed analytically, reducing to the solution of a

linear system of equations, one equation for each layer thickness l^i . Then, the temperatures and densities are varied and the process repeated until the set of temperatures, densities, and layer thicknesses is found which minimizes Q^2 for a given number of layers, N . This set is referred to as the best N -layer fit. This procedure can be repeated for models with differing numbers of layers until a sufficiently accurate fit is found.

The average temperature and density in the absorbing region can be inferred from this best fit. For example, the inference for the average temperature from the best N -layer fit would be

$$\bar{kT}_N = \frac{\sum_{i=1}^N kT^i l^i}{\sum_{i=1}^N l^i}.$$

The existence of temperature and/or density gradients in the absorbing region can be inferred when the best one-layer fit does not yield the lowest Q^2 . The magnitude of these gradients can only be estimated if one has some knowledge of the temperature and density profiles (*e.g.*, that they are monotonic). Note that the model used here does not distinguish between different orderings of the same set of layers. With an assumption as to the ordering, an inference for the average density gradient would be

$$\bar{n}_{eN} = \frac{n_e^N - n_e^1}{\sum_{i=1}^N l^i}.$$

To explore the capabilities of this model, this chapter presents the analyses of two synthetic spectra. These spectra were produced using the model discussed above for absorption through a region with temperature and/or density gradients. In both cases, $I_0(\nu)$ was a constant in the relevant spectral region and both spectra were convolved with a Gaussian instrumental function of FWHM 0.1 eV, chosen to practically eliminate the effect of instrumental

broadening. Thus, the analyses of these two spectra were conducted to test the performance of this method of analysis under idealized conditions.

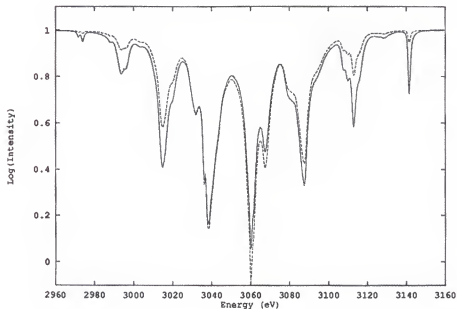


Figure 26: First synthetic spectrum (solid) and the best one- (long dashes), two- (short dashes), and three- (dots) layer fits.

The first synthetic spectrum simulates absorption by a uniform density plasma with a temperature gradient. This spectrum, and the best one-, two-, and three-layer fits are shown in Figure 26. Values of the relevant plasma parameters for this spectrum and the best one- and two-layer fits are given in Table 5.

All three best fits were produced by absorbing regions with the same density as the synthetic spectrum. In this simple case, the quality of the fits improve substantially with increasing N up to 3. Also shown are plots of Q^2 vs. the

temperature of one of the layers holding the temperature of the other layers fixed at their values in the best fits (Figure 27).

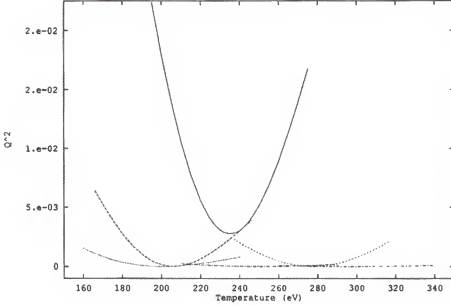


Figure 27: Q^2 vs. kT in one of the fitting layers, holding the other layers fixed at the value for the best fit for the one-layer fit (solid), the two-layer fit (long dashes, short dashes), and the three-layer fit (dots, long dash-dot, short dash-dot).

Figures 26 and 27 reveal both the strength and weakness of this method of analyzing absorption spectra. The narrow parabolic shape of the dependence of Q^2 upon kT for the one-layer fit allows for an easy identification of the best one-layer fit. Increasing N to 2 allows for a lower Q^2 but decreases the sensitivity of Q^2 to variations in the temperatures of the two layers in the fitting spectrum. When N is increased to 3, the value of Q^2 goes effectively to zero (the synthetic spectrum only had three layers) but the sensitivity of Q^2 to variations in the

layer temperatures decreases dramatically. While for the noise-free, idealized synthetic spectra considered here this decrease in sensitivity did not affect the results of the analysis, this behavior would place a practical limit on the number of layers used to fit an actual experimental spectrum.

Table 5: Parameters for the first synthetic spectrum and the best one- and two-layer fits.

First	Synthetic		Spectrum
Layer l (μm)	kT (eV)	n_e (10^{24}cm^{-3})	
1	0.5	200	1.0
2	0.3	250	1.0
3	0.1	300	1.0
	$\sum l$	$k\bar{T}$	$\bar{n}_e$
	0.9	228	1.0
Best One Layer Fit ($Q^2 = 2.8 \times 10^{-3}$)			
1	0.8	235	1.0
Best Two Layer Fit ($Q^2 = 1.1 \times 10^{-5}$)			
1	0.6	206	1.0
2	0.3	277	1.0
	$\sum l$	$k\bar{T}$	$\bar{n}_e$
	0.9	228	1.0

The second synthetic spectrum simulates absorption by a significantly more complex plasma with non-negligible temperature and density gradients. The spectrum and the best 1-, 2-, and 3-layer fits are shown (Figure 28) and the values for the relevant plasma parameters tabulated (Table 6). Unlike the best one-layer fit for the first spectrum, the best one-layer fit for this spectrum does not produce accurate inferences for any of the plasma parameters. However,

the best 2- and 3-layer fits yield increasingly accurate inferences for the plasma parameters and visibly improving fits. The dependence of Q^2 upon the fitting parameters for this synthetic spectrum is qualitatively similar to that of the first synthetic spectrum.

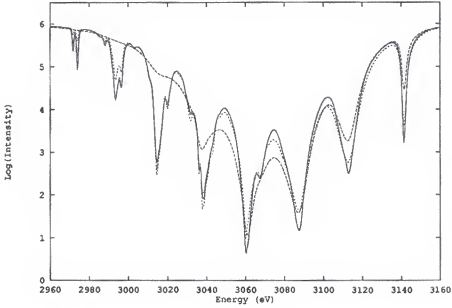


Figure 28: Second synthetic spectrum (solid) and the best one- (long dashes), two- (short dashes), and three- (dots) layer fits.

Recapitulation

The broadening properties of $n = 1$ to $n = 2$ inner shell transitions in the L-shell ions of Ar were studied, and it was found that Stark broadening is important in the 0.5 to $5 \times 10^{24} \text{cm}^{-3}$ electron number density range, at temperatures between 100 and 600 eV. Absorption line profiles were computed

Table 6: Parameters for the second synthetic spectrum and the best one- and two-layer fits.

Second		Synthetic	Spectrum
Layer l (μm)	kT (eV)	n_e (10^{24}cm^{-3})	
1	1.0	130	0.5
2	1.0	150	0.5
3	0.5	180	0.5
4	0.6	200	1.0
5	0.6	220	1.0
6	0.5	300	2.0
7	0.3	350	2.0
8	0.8	400	5.0
$\sum l$		$k\bar{T}$	$\bar{n}_e$
5.3		233	1.5
Best One Layer Fit ($Q^2 = 2.8 \times 10^{-3}$)			
1	1.7	407	5.0
Best Two Layer Fit ($Q^2 = 1.1 \times 10^{-5}$)			
1	3.5	170	0.5
2	1.3	439	5.0
$\sum l$		$k\bar{T}$	$\bar{n}_e$
4.7		241	1.7
Best Three Layer Fit ($Q^2 = 1.1 \times 10^{-5}$)			
1	2.9	153	0.5
2	1.5	298	2.0
3	0.7	465	5.0
$\sum l$		$k\bar{T}$	$\bar{n}_e$
5.1		238	1.5

for all the L-shell Ar ions and used to calculate the frequency-dependent optical depth under high-density conditions. This optical depth incorporates the

temperature and density dependence of the population distribution and the density dependence of the Stark-broadened absorption line profiles.

To explore the diagnostic capabilities of this temperature and density sensitivity, preliminary results based on the analyses of synthetic spectra were presented. The analyses of these two synthetic spectra reveal general characteristics of the model. As N increases, the quality of the fit, as measured by Q^2 improves, while the sensitivity of the fit to changes in the fitting parameters decreases. This behavior sets a practical limit on the number of layers that can be used to fit a given spectrum. The utility of this method of analysis thus depends on whether the quality of the fit approaches an acceptable level and the inferred values of the plasma parameters of interest converge while the sensitivity of Q^2 to changes in those parameters is sufficient to provide reliable inferences.

CHAPTER 5

STARK BROADENING THEORY APPLIED TO CONTINUUM RADIATOR WAVEFUNCTIONS

The preceding chapters concentrated on the use of Stark broadened line profiles in analyses of emission and absorption spectra. Integral to the analyses was the inclusion of satellite transitions whose upper states are susceptible to the process of autoionization. In Chapter 4, the effect of autoionization on the lineshapes was approximated by convolving the Stark broadened lineshapes with a Lorentzian whose width was dictated by the autoionization rates, following the work of Davis and Jacobs⁵⁷. This approach dramatically fails in the isolated atom case, due to the interference between the processes of autoionization and radiative decay which leads not to a slightly broader line, but to a dramatically asymmetric Fano profile. Yet, the simple convolution approach leads to lineshapes which fit the data well. Somewhere between the isolated radiator and the radiator immersed in a dense plasma, the effects of this interference are somehow mitigated. This chapter develops an appropriate theoretical approach to the Stark Broadening of transitions involving continuum radiator wavefunctions.

Two issues arise when Stark Broadening Theory is applied to transitions involving continuum radiator wavefunctions, one technical, the other more fundamental. The technical issue stems from the need to invert resolvents operating on continuum radiator wavefunctions. The more fundamental issue concerns the inapplicability of truncated multipole approximations in cases where the

radiator wavefunction is not localized. This chapter recapitulates the development of the Stark Broadening Theory, places these two issues within their context and illustrates how they can be dealt with within the framework of that highly successful theory.

Return to the expression for the lineshape developed in Chapter 2, expressed as the Fourier transform of the radiator's dipole-dipole time autocorrelation function:

$$I(\omega) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{rei} [\vec{d} \cdot \vec{d}(t) \rho_{rei}]$$

Again using the Liouville space formalism to treat the time development of the system, with the Liouville operator L defined by

$$L\Omega = [H, \Omega] \quad , \quad \vec{d}(t) = e^{-\frac{iLt}{\hbar}} \vec{d}.$$

Again, use the static ion approximation and factorize the initial density matrix,

$$\vec{d}(t) = e^{-\frac{iL_{eet}t}{\hbar}} \vec{d} \quad , \quad \rho_{rei} = \rho_{re} \rho_i.$$

The lineshape in this approximation is

$$I(\omega) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{rei} [\vec{d} \cdot e^{-\frac{iL_{eet}t}{\hbar}} \vec{d} \rho_{re} \rho_i].$$

Recall that $H_{e,r} = H_e + H_r$, and thus $L_{e,r}$, depend on the perturbing ion coordinates through $V_{i,r}$. What if $V_{i,r}$ did not depend on the ion coordinates simply through their combined electric field at the nucleus of the radiator? Approximating the radiator-perturbing ion interaction potential by the first two terms of its multipole expansion is appropriate only when the size of the radiator is small compared with the length scale associated with variations in the ion microfield. Indeed, the validity of any truncation of the multipole

expansion depends on there being a point in the expansion beyond which the terms are negligible: when there exists an n such that $\langle r \rangle^m / r_i^m$ is small for all m greater than n , where $\langle r \rangle$ is the size of the radiator, and r_i is the average inter-ion spacing. For continuum radiator wavefunctions, for which there are no appropriate natural length scales, a truncated multipole approximation is thus inappropriate. Mindful of this, I still discuss the ion microfield and microfield constrained averaging of operators, recognizing that a different approximation to $V_{i,r}$ will be required.

Microfield Constrained Averages

For an operator Ω depending on the perturbing ion coordinates ($\Omega = \Omega(\{\vec{R}_{\text{ions}}\})$) the microfield constrained average of Ω is given by

$$\langle \Omega \rangle_{\vec{E}} = \frac{\text{Tr}_i \rho_i \Omega \delta(\vec{E}(\{\vec{R}_{\text{ions}}\}) - \vec{E})}{\text{Tr}_i \rho_i \delta(\vec{E}(\{\vec{R}_{\text{ions}}\}) - \vec{E})}.$$

The denominator in this expression is just the static ion microfield distribution function, $Q(\vec{E})$, here calculated in the APEX approximation of Iglesias, Lebowitz, and MacGowan²². It is convenient to introduce the constrained ion pair correlation function, $g(\vec{R}; \vec{E})$, given by

$$\langle n(\vec{R}) \rangle_{\vec{E}} = n_i g(\vec{R}; \vec{E}),$$

since for an operator Ω which depends on the ion coordinates only through their pair-wise interaction with the radiator,

$$\Omega = \sum_{\text{ions}} \omega(\vec{R}_i),$$

its constrained average is given by

$$\langle \Omega \rangle_{\vec{E}} = n_i \int d\vec{R} g(\vec{R}; \vec{E}) \omega(\vec{R});$$

then the expectation value of Ω can be calculated by a microfield integration:

$$\text{Tr} \rho_i \Omega = \int d\vec{E} Q(\vec{E}) \langle \Omega \rangle_{\vec{E}}.$$

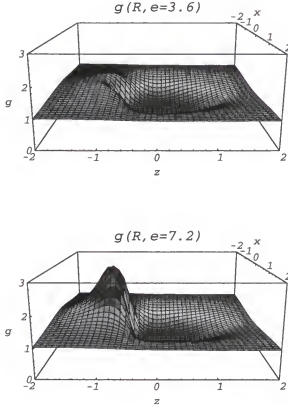


Figure 29: $g(\vec{R}; \vec{E})$ for two different values of $\vec{E}$, for a He-like Ne plasma with $n_e = 5 \times 10^{22} \text{cm}^{-3}$ and a temperature of 800eV, shown in the x-z plane. The function is cylindrically symmetric about the z-axis. The field is given in units of e/r_{0e}^2 , and distance in r_{0i} .

Figure 29 displays $g(\vec{R}; \vec{E})$ for two different values of $\vec{E}$. $\vec{E}$ in the positive z direction implies an overall concentration of ions in the negative z direction. As the magnitude of the field at the radiator increases, this concentration becomes more peaked around one average ion spacing from the radiator. The

calculation of the constrained pair correlation function is performed using the method of Lado and Dufty⁵⁸, in the APEX approximation.

Using the formalism of microfield constrained averages, one can cast the lineshape in a familiar form

$$I(\omega) = \int d\vec{E} Q(\vec{E}) J(\omega; \vec{E}),$$

with the single ion microfield electron broadened lineshape now given by

$$J(\omega; \vec{E}) = \langle \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot e^{-\frac{iL_{gr}t}{\hbar}} \vec{d} \rho_{re}] \rangle_{\vec{E}}.$$

Radiator-Plasma Ion Interaction

Having cast the lineshape in this familiar form, I now turn to the calculation of the single ion microfield electron broadened lineshape, $J(\omega; \vec{E})$. As the plasma ion coordinates only occur in the time development operator, the microfield constrained averaging only effects this operator, and one must calculate

$$\langle \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot e^{-\frac{iL_{gr}t}{\hbar}} \vec{d} \rho_{re}] \rangle_{\vec{E}} = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{re} [\vec{d} \cdot \langle e^{-\frac{iL_{gr}t}{\hbar}} \rangle_{\vec{E}} \vec{d} \rho_{re}].$$

In the Liouville operator, the ion coordinate dependence comes through the radiator-plasma ion interaction with the monopole term subtracted out, having been included in the microfield distribution function calculation.

$$\begin{aligned} V_{i,r}^1 &= V_{i,r} - \text{monopole term} \\ &= -\vec{E} \cdot \vec{D} - \frac{1}{6} Q_{\mu,\nu} \delta_{\mu} E_{\nu} + \dots \end{aligned}$$

Some type of multipole expansion, like that in the second line in the previous equation, is always exact, but is not always useful. (Kilcrease and Murrilo discuss the details associated with the non-zero screening charge at the

radiator⁵⁹.) The utility of a multipole expansion rests on its ability to be truncated for the particular problem of interest. For the case of Stark broadening of bound-bound transitions, truncation after the dipole term has often been used^{13,12}, with occasional authors exploring the effect of the quadrupole term^{60,61,62}.

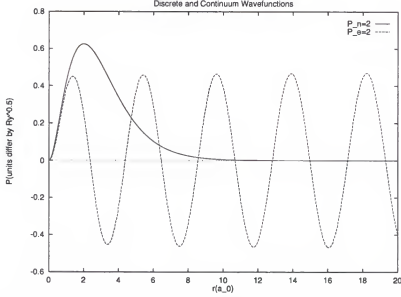


Figure 30: An illustration of the spatial dependence of bound and continuum wavefunctions. Note that the units perform differ by a factor of $Ry^{1/2}$.

These truncations rely on the smallness of the expansion parameter, the ratio of the length scale associated with the radiator, its “size” $\langle r \rangle$, to the length scale associated with derivatives of the potential, the average inter-ion spacing r_{0i} . Increasing orbital size or plasma density can thus necessitate the use of more terms in the expansion. Kilcrease, Mancini, and Hooper⁶² found that, for conditions and bound-bound transitions relevant to the analysis of ICF plasmas, the effect of the quadrupole term was small, at least for conditions where the underlying non-degenerate electron broadening theory was applicable.

When Stark broadening theory is applied to transitions involving continuum radiator wavefunctions, a truncation of the multipole expansion is inappropriate, as there is no length scale associated with the radiator electrons' wavefunctions. Indeed, continuum radiator wavefunctions sample the plasma ion microfield everywhere. I proceed to introduce a new approximation for the radiator-plasma ion interaction which retains the calculational simplicity of the ion microfield formalism, reduces to the dipole approximation for tightly bound radiator states, and approximately accounts for the spatial variation of the plasma ion microfield.

The radiator-plasma ion interaction with the monopole term removed, V_{ir}^1 , can be expressed as

$$V_{ir}^1 = \int d\vec{r} \rho(\vec{r}) [V(\vec{r}; \{\vec{R}_{\text{ions}}\}) - V(\vec{0}; \{\vec{R}_{\text{ions}}\})],$$

where, in a convention to be followed in the rest of this work, the lower case $\vec{r}$ is the radiator electron coordinate, and the capital $\vec{R}$ is a plasma ion location. $\rho(\vec{r})$ is the charge density of the radiator (classically, the quantum generalization being the electron charge multiplied by the radiator electron density matrix), and $V(\vec{r}; \{\vec{R}_{\text{ions}}\})$ is the potential at $\vec{r}$ due to all the plasma ions. The difference between the scalar quantity V at $\vec{r}$ and $\vec{0}$ can be written as an integral over the appropriate gradient:

$$V_{ir}^1 = \int d\vec{r} \rho(\vec{r}) \vec{r} \cdot \int_0^1 d\lambda \nabla_{\lambda \vec{r}} V(\lambda \vec{r}; \{\vec{R}_{\text{ions}}\}).$$

This naturally leads to the definition of an effective ion electric field, $\vec{F}(\vec{r}; \{\vec{R}_{\text{ions}}\})$,

$$\vec{F}(\vec{r}; \{\vec{R}_{\text{ions}}\}) = - \int_0^1 d\lambda \nabla_{\lambda \vec{r}} V(\lambda \vec{r}; \{\vec{R}_{\text{ions}}\}),$$

allowing us to cast the radiator-plasma ion interaction in the familiar form

$$V_{ir}^1 = - \int d\vec{r} \rho(\vec{r}) \vec{r} \cdot \vec{F}(\vec{r}; \{\vec{R}_{\text{ions}}\}).$$

Using this effective electric field, the microfield constrained average of V_{ir}^1 can be written

$$\langle V_{ir}^1 \rangle_{\vec{E}} = - \int d\vec{r} \rho(\vec{r}) \vec{r} \cdot \langle \vec{F}(\vec{r}) \rangle_{\vec{E}}.$$

To compare this to the usual dipole approximation, write

$$\langle V_{ir}^1 \rangle_{\vec{E}} = - \left[\int d\vec{r} \rho(\vec{r}) \vec{r} \cdot \vec{E} - \int d\vec{r} \rho(\vec{r}) \vec{r} \cdot (\langle \vec{F}(\vec{r}) \rangle_{\vec{E}} - \vec{E}) \right].$$

From the definition of the effective field, $\langle \vec{F}(\vec{0}) \rangle_{\vec{E}} = \vec{E}$, and thus this approximation to V_{ir}^1 reduces to the dipole approximation for tightly bound states: excursions from the dipole approximation occur whenever the spatially varying effective field differs from the ion microfield value in regions where the radiator charge density is not negligible. Figure 31 shows an example of the effective field, and illustrates the utility of the truncation of the multipole expansion for tightly bound states, and the reason why such a truncation is inappropriate for continuum radiator wavefunctions.

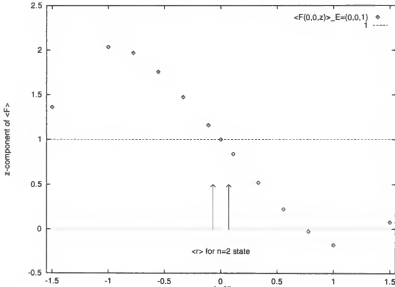


Figure 31: Microfield constrained effective field $\langle \vec{F}(\vec{r}) \rangle_{\vec{E}}$ for the plasma of Figure 1 along the z -axis.

A Different Truncation

Given the inappropriateness of truncation of a multipole expansion for transitions involving continuum radiator wavefunctions, a different sort of truncation is needed, and I will truncate a cumulant expansion of the microfield constrained time development operator and comment on the first correction term. Recall that the single ion microfield electron broadened lineshape, $J(\omega; \vec{E})$ was expressed as

$$J(\omega; \vec{E}) = \frac{1}{\pi} \Re \int_0^\infty dt e^{i\omega t} \text{Tr}_{r,e} \vec{d} \cdot \langle e^{-i\frac{L}{\hbar}t} \rangle_{\vec{E}} \vec{d} \rho_{r,e}.$$

A cumulant expansion of the microfield constrained time development operator yields:

$$\langle e^{-i\frac{L}{\hbar}t} \rangle_{\vec{E}} = e^{-\frac{i\omega}{\hbar} \langle L \rangle_{\vec{E}} - \frac{t^2}{2\hbar^2} \langle (L - \langle L \rangle_{\vec{E}})^2 \rangle_{\vec{E}} + \dots}.$$

The second order term is related to

$$\begin{aligned} & \langle (V_{ir}^1 - \langle V_{ir}^1 \rangle_{\vec{E}})^2 \rangle_{\vec{E}} \\ &= \int d\vec{r} \rho(\vec{r}) \int d\vec{r}' \rho(\vec{r}') \int d\vec{R} \int d\vec{R}' v(\vec{r}, \vec{R}) v(\vec{r}', \vec{R}') [g(\vec{R}, \vec{R}'; \vec{E}) - g(\vec{R}; \vec{E}) g(\vec{R}'; \vec{E})], \end{aligned}$$

and thus truncation of the cumulant expansion after the first term is appropriate for those plasmas where the three point function can be approximated as the product of the related two point functions. The method is thus limited to moderately coupled plasmas. It is interesting to note that the APEX approximation is equivalent to re-writing the radiator-plasma ion and the ion-ion interactions as pseudoparticles interacting only with the radiator with a screening length chosen to satisfy two exact moment conditions for the microfield at the radiator, and that, for such pseudoparticles, the unconstrained three point function is given by the product of the two point functions. Joyce, Woltz, and Hooper⁶¹ and Kilcrease, Mancini, and Hooper⁶² used a similar

truncation for the time development operator for their work on the effect of the quadrupole term on bound-bound transitions, though their truncation was justified by the limited spatial extent of the radiator wavefunctions in which they were interested.

In this truncated cumulant expansion approximation, then, the lineshape is given by

$$J(\omega; \vec{E}) = \frac{1}{\pi} \Re \text{Tr}_{r,e} [\vec{d} \cdot \int_0^\infty dt e^{it(\omega - \frac{\langle L \rangle \vec{E}}{\hbar})} \vec{d} \rho_{re}],$$

or, performing the integration

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \text{Tr}_r [\vec{d} \cdot R_r(\omega; \vec{E}) \vec{d} \rho_r].$$

The resolvent is thus approximated as

$$R_r(\omega; \vec{E}) = \frac{1}{\omega - \frac{L_r^0}{\hbar} - \frac{\langle L \rangle \vec{E}}{\hbar} - \phi_e}.$$

As I am here concerned primarily with the effects of the ion microfield, I will choose a simple impact approximation for the electron broadening operator². The microfield constrained electron broadening operator has been previously considered¹⁹ and will be the subject of future investigations.

The next chapter discusses the application of the methods developed in this chapter to illustrative model systems.

CHAPTER 6 ILLUSTRATIVE MODEL SYSTEMS

In order to illustrate the application of Stark broadening theory to transitions involving continuum radiator wavefunctions I calculate the lineshapes from two model systems designed to focus narrowly on the novel aspects of these calculations while avoiding the computational complications introduced by real world atomic physics. The first will illustrate the calculation of the resolvent by the inversion of operators acting on continuum wavefunctions, and the second will illustrate the effect of the spatial variation of the microfield on the emission from an autoionizing transition.

Calculation of the Resolvent

To focus on the calculation of the resolvent, consider the model system detailed in Figure 32, a bound-bound hydrogenic transition perturbed by a continuum.

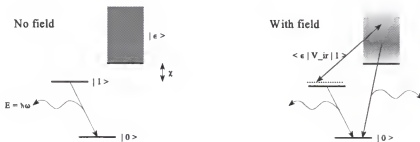


Figure 32: Depiction of the model system discussed in this section, the $2p \rightarrow 1s$ transition in hydrogen perturbed by the ϵd ($\ell = 2$) continuum through the action of the plasma microfield.

This model system also shows how the plasma microfield can mix continuum states with bound states, and how this mixture causes a distortion in the emission above the ionization potential. In this case, all the non-vanishing matrix elements of $R^{-1}(\omega; \vec{E})$ involve at least one bound state, and thus the dipole approximation to V_{ir}^1 will be used. In the absence of the plasma ion microfield there would be a single electron broadened peak at $\hbar\omega_{10}$. The ion microfield mixes the upper state of the transition with the perturbing continuum, dressing the continuum and modifying the lineshape.

Calculation of the resolvent requires the inversion of a combination of known operators,

$$R^{-1}R = \mathcal{I}.$$

For this system, the appropriate resolution of unity is

$$\mathcal{I} = |0\rangle\langle 0| + |1\rangle\langle 1| + \int d\varepsilon |\varepsilon\rangle\langle \varepsilon|.$$

For the lineshape, I will require only one component of R , $\langle 1|R|1\rangle$:

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \langle 0|\vec{d}|1\rangle \langle 1|R_r(\omega; \vec{E})\rho_r\vec{d}|0\rangle.$$

Of the nine components of the matrix equation $R^{-1}R = \mathcal{I}$, the desired component of R appears in only 3, and those 3 suffice for the calculation of that matrix element.

$$\begin{aligned} \langle 0|\mathcal{I}|1\rangle &= 0 = \langle 0|R^{-1}|0\rangle\langle 0|R|1\rangle + \langle 0|R^{-1}|1\rangle\langle 1|R|1\rangle + \int d\varepsilon'' \langle 0|R^{-1}|\varepsilon''\rangle\langle \varepsilon''|R|1\rangle \\ \langle 1|\mathcal{I}|1\rangle &= 1 = \langle 1|R^{-1}|0\rangle\langle 0|R|1\rangle + \langle 1|R^{-1}|1\rangle\langle 1|R|1\rangle + \int d\varepsilon'' \langle 1R^{-1}|\varepsilon''\rangle\langle \varepsilon''|R|1\rangle \\ \langle \varepsilon|\mathcal{I}|1\rangle &= 0 = \langle \varepsilon|R^{-1}|0\rangle\langle 0|R|1\rangle + \langle \varepsilon|R^{-1}|1\rangle\langle 1|R|1\rangle + \int d\varepsilon'' \langle \varepsilon|R^{-1}|\varepsilon''\rangle\langle \varepsilon''|R|1\rangle. \end{aligned}$$

After carrying the analytical solution as far as possible, one finds

$$\langle 0|R|1\rangle = 0$$

$$\langle 1|R|1 \rangle = \frac{1}{(\omega - \omega_{10} + i\Phi)} + \int d\epsilon'' \frac{\langle 1|\vec{E} \cdot \vec{D}|\epsilon'' \rangle \langle \epsilon''|R|1 \rangle}{(\omega - \omega_{10} + i\Phi)}$$

$$\frac{\langle \epsilon|\vec{E} \cdot \vec{D}|1 \rangle}{(\omega - \omega_{10} + i\Phi)} = \int d\epsilon'' \left[(\omega - \omega_{\epsilon 0} + i\Phi) \delta(\epsilon - \epsilon'') - \frac{\langle \epsilon|\vec{E} \cdot \vec{D}|1 \rangle \langle 1|\vec{E} \cdot \vec{D}|\epsilon'' \rangle}{(\omega - \omega_{10} + i\Phi)} \right] \langle \epsilon''|R|1 \rangle.$$

The second term in the last integral explicitly shows the ion microfield mixing the initial bound state with the perturbing continuum. To calculate the lineshape, I need $\langle 1|R|1 \rangle$, which in turn requires the solution of an integral equation for $\langle \epsilon''|R|1 \rangle$. This equation has the form

$$f_{\omega, \vec{E}}(\epsilon) = \int d\epsilon'' g_{\omega, \vec{E}}(\epsilon, \epsilon'') h_{\omega, \vec{E}}(\epsilon''),$$

where f and g are known functions. The strongly diagonal nature of g motivates an attempt to solve the equation by discretizing the continuum:

$$f_i = \sum_j (g_{ij} \Delta_j) h_j,$$

with the size of the mesh determined by the convergence of the procedure.

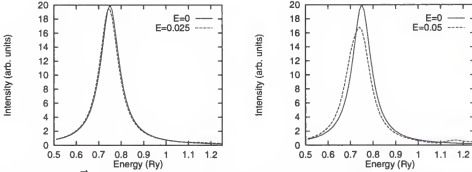


Figure 33: $J(\omega, \vec{E})$ for the model system discussed in this section, for three different values of the microfield, $\vec{E}$, given in atomic units.

The effect of mixing of the initial state and the continuum through the ion microfield discussed above is demonstrated in Figure 33, which displays two single-ion-microfield electron-broadened lineshapes, for different values of the

ion microfield. As the microfield increases, the mixing of the continuum and bound state increases, and with it the redshift of the line and the enhancement of intensity above the continuum edge.

An Auto-ionizing Transition

The two-electron radiator system pictured in Figure 34 is designed to highlight the effect of continua interacting through the spatially varying plasma ion microfield. In the absence of the configuration interaction there are two qualitatively different types of states above the first ionization potential, namely doubly excited bound states, and states with one bound electron in the ground state, and an unbound electron, whose angular quantum numbers are used to discriminate between different continua. When the configuration interaction is accounted for, this degeneracy is removed, and the remnants of the doubly excited states are included in auto-ionizing continua.

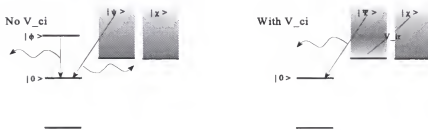


Figure 34: Depiction of the model system discussed in this section, a transition from an autoionizing continuum to a bound state, perturbed by a continuum through the action of the plasma microfield.

Fano⁶³ has elegantly calculated the coefficients of the resulting admixture, and the resulting eponymous asymmetric profiles. In this model system, the single autoionizing continuum forms the initial states from which the dipole

transitions occur and which is perturbed by the adjacent continuum through the action of the plasma ion microfield.

To be specific: I used as the unperturbed states properly symmetrized product wavefunctions of hydrogenic orbitals for a Ne ($Z = 10$) nucleus, and assumed for plasma conditions an electron density of $5 \times 10^{22} \text{cm}^{-3}$, and a temperature equivalent to 800eV. The final state, $|0\rangle$ is the $1s2s$ state, and the autoionizing continuum, $|\Psi\rangle_\varepsilon$ is an admixture of $|\phi\rangle$, the $2s2p$ state, and $|\psi\rangle_\varepsilon$, the $1\varepsilon p$ continuum, which admixture is given by the expression

$$|\Psi\rangle_\varepsilon = a(\varepsilon)|\phi\rangle + \int d\varepsilon' b(\varepsilon, \varepsilon') |\psi'_\varepsilon\rangle \langle\psi_\varepsilon|,$$

where the coefficients a and b are calculated by the method of Fano⁶³. The perturbing continuum, $|\chi\rangle$, is $1\varepsilon d$.

For this system, the appropriate resolution of unity is

$$\mathcal{I} = |0\rangle\langle 0| + \int d\varepsilon [|\Psi_\varepsilon\rangle\langle\Psi_\varepsilon| + |\chi_\varepsilon\rangle\langle\chi_\varepsilon|],$$

and the single ion microfield electron broadened lineshape is given by

$$J(\omega; \vec{E}) = -\frac{1}{\pi} \Im \int d\varepsilon \int d\varepsilon' \vec{d}(\varepsilon) \cdot \vec{d}(\varepsilon') R(\omega; \vec{E})_{\varepsilon, \varepsilon'} \rho(\varepsilon).$$

As in the previous model system, the calculation of the resolvent involves the solution of an integral equation whose solution is facilitated by the strongly diagonal nature of the kernel of the integration. As the effects of the ion microfield would be obscured by electron broadening, for purposes of illustration we arbitrarily set the electron broadening to 5% of the value calculated in the impact approximation. The results are displayed in Figure 35.

The left hand plot in Fig. 35 shows that even this small amount of electron broadening is sufficient to remove the dip characteristic of isolated atom

Fano profiles for this transition with its large Fano q parameter.⁶³ The right hand plot shows the effect on the peak of increasing the ion microfield. For the autoionizing transitions currently used in spectral plasma diagnostics⁶⁴, in the current range of plasma conditions, the procedure of Woltz, Jacobs, Hooper, and Mancini⁶⁵, namely convolving the bound-bound lineshape with a Lorentzian whose width is given by the autoionizing rate is expected to be adequate. Indeed, the autoionizing width in this model is so small that other effects, such as ion dynamics and other decay channels would need to be considered if one were interested in the calculation of such lineshapes for diagnostic purposes.

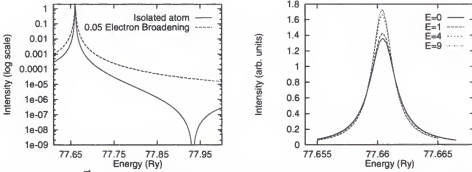


Figure 35: $J(\omega; \vec{E})$ for the system of Figure 3, for four different values of the microfield, given in average inter-ion field units. On the left, the effect of electron broadening on the dip of the Fano profile is shown (not the log. scale). On the right, the effect of increasing microfield on the peak is displayed.

Comments

The method and models presented here demonstrate that, in principle, Stark broadening theory can be applied to transitions involving continuum radiator wavefunctions, at least for those plasmas where the coupling permits the truncation of the cumulant expansion of the time-development operator.

While the models have served their purpose as a proving ground for the method, they are a bit abstract, and in this section I will speculate about the application of Stark broadening theory to problems of current interest.

Stark broadening theory can be thought of as answering the question “How does the plasma environment effect the process of radiative decay?” Now that a method exists for the inclusion of continuum states in Stark broadening calculations, other processes can be addressed within the same formalism, including those processes present in rate matrices used in the calculation of population distributions. Such ion density dependent rate matrices would be of interest for dense plasmas, where the use of isolated atom rates seems counter-intuitive.

Related to the question of the calculation of level populations in dense plasmas is the question of continuum lowering. Keane, et al.⁹ have shown that the inclusion of a particular model of continuum lowering greatly effects calculated spectral modeling of emission from ICF implosions. The problem of ion-microfield-induced continuum lowering seems well suited to Stark broadening analysis, as the effect is one where the plasma ion microfield mixes bound radiator states and unbound radiator states.

The canonical picture of continuum lowering does not take into account the spatial variation of the plasma ion microfield, and is thus suspect. Perhaps the spatially dependent $\langle V_{ir}^1(\vec{r}) \rangle_{\vec{E}}$ introduced here would be more appropriate for the mixing of the spatially extended high-lying bound states and the nearby continua. This field point of view can be seen as the obverse of the multi-center radiator point of view espoused by Gauthier⁶⁶ and Foulis, et al.⁶⁷ and comparison of the results of these two complementary approaches would

be interesting. A note of caution is, perhaps, warranted. The time scales associated with free-free and free-bound processes will require a re-examination of the approximations used for electron broadening and ion screening.

To conclude, Stark broadening theory can treat transitions involving continuum radiator wave functions, within the coupling limits imposed by the truncation of the cumulant expansion of the time-development operator. Such treatment requires the inversion of operators acting on continuum states, the inversion of which is simplified by the strongly diagonal nature of the operators. The effects of the spatial variation in the plasma ion microfield on these spatially extended states can be approximately accounted for through the use of microfield constrained operators, and the calculation of these operators is greatly facilitated by use of the technique of Lado and Dufty⁵⁸. This work opens the door for the application of the highly successful theory of Stark broadening to new and interesting problems, such as ion density dependent rate matrices and continuum lowering.

CHAPTER 7 CONCLUSIONS AND DISCUSSION

In this dissertation, theoretical spectra have been developed for the purpose of inferring environmental conditions from the spectral radiation emitted by hot dense plasmas. These theoretical spectra have included contributions from auto-ionizing satellites, transitions which involve continuum- as well as discrete-radiator wavefunctions. Techniques and approximations appropriate for the application of Stark broadening theory to such transitions have been introduced, and illustrated by the calculation of spectra from model systems constructed to highlight novel aspects of the problem.

The ICF environment is not an ideal one for the benchmarking of Stark broadening calculations. The transience of the plasma, as well as the lack of assured implosion symmetry, make the interpretation of spectra from ICF implosions non-trivial. If one were interested solely in checking the results of Stark broadening theory against experiment, one would turn to more tame experimental configurations, such as gas-liner pinch discharges⁶⁸, where the plasmas last a thousand times longer than typical ICF plasmas, though they are considerably cooler (around 50 eV as opposed to 1keV), and less dense by six orders of magnitude. The advantage of this type of experiment is that the electron density can be confirmed by means which do not depend on line broadening analysis, such as Thompson scattering.⁶⁹ The disadvantages are at least two-fold. The low densities and temperatures force the use of low-Z, non-relativistic radiators, thus rendering the effects of fine structure difficult

to assess. Also, it seems unlikely that using such experimental procedures will produce plasma conditions where interesting physical effects, such as electron degeneracy and continuum lowering, will be obtained. Thus, there are reasons for pursuing ICF-type implosions.

Spatial inhomogeneities are endemic to present day implosions. X-ray images of implosions frequently show elliptical cores, though they occasionally take on more exotic forms. (In the most extreme case I have seen, the image from a recent shot at LLE would likely elicit the response "three-leaf clover" in a Rorschach test). Yet, the results of Figure 17 are typical, namely that a single temperature and density often suffice for the fitting of the prominent features of a given emission spectrum. Given that the spectrum emitted by a plasma is emissivity averaged, it is likely that there is a fairly uniform part of the core which is the dominant source of emission. If this is the case, then x-ray line spectroscopy, as currently used, will inform only about conditions in this core, and not of the plasma as a whole. However, the recent availability of time-resolved x-ray monochrometry^{70,71} has allowed the beginnings of investigations such as whether or not the Li-like satellites are emitted from the same region of the plasma as their associated He-like resonance lines, and thus the extent to which composite spectral features formed by satellites and resonance lines are truly useful diagnostic tools.

Another encouraging development is the availability of advanced nuclear diagnostics at the University of Rochester's Laboratory for Laser Energetics.⁷² Though few of the nuclear instruments are time-resolved, and they are neutron-, rather photon-emissivity averaged, the information about the implosion from both nuclear and spectroscopic diagnoses will provide complementary information.

An experimental program using both x-ray monochrometry and nuclear diagnostics to supplement Stark broadening analysis of implosions has been approved and will be conducted over the next year, and the analysis of the resulting data should allow further statements about the effect of spatial inhomogeneities on spectra.

The analysis of Chapter 3 does not suffer appreciably from the admitted likely existence of spatial inhomogeneities in the plasma, so long as it is admitted that the inferences are likely to characterize only the most emissive part of the plasma. Further, the effects of opacity alone cannot explain the lack of dip at the center of the He- β line in the data in Figure 15, nor can ion dynamics alone account for the lack of a dip in the data from Figure 16. An essential point of Chapter 3, that ion dynamics and opacity effects generally must both be included in the analysis of plasmas where just a small amount of dopant has been added to the D₂ fuel is illustrated when those two figures are considered together.

The theoretical development of Chapter 4 is straightforward, and the resulting calculated absorption features have proved useful in the analysis of spectra from implosions where the fill gas was pure Ar.⁵² However, the inferred sub-micron geometrical lengths are counter-intuitive, as there seems no reason why, in an implosion of a microballoon of 420 μ m diameter, with a 6 μ m thick shell, should result in a sub-micron thick high density, low temperature absorbing region. However, when it is noted that the high pressure (20 Atm) Ar filled capsules are susceptible to diffusion of the Ar into the shell, it seems possible that the absorbing region may come from Ar in the compressed residual shell. More experimental work is clearly needed in this area, and it, too, is expected to be performed in the near future.

The model systems of Chapter 6, while they do serve their purpose as illustrations of the techniques and approximations developed in Chapter 5, are highly abstract, and perhaps it would be appropriate here to discuss further the possible practical applications of those techniques and approximations.

When the lineshape is written as the Fourier transform of a dipole-dipole time autocorrelation function,

$$I(\omega) = \int dt e^{i\omega t} \text{Tr}[\vec{d} \cdot \vec{d}(t)\rho],$$

there are a large number of possible systems to which the formalism can be applied. Depending on the detailed meaning chosen for the operators $\vec{d}$ and ρ , the expression is appropriate for bound-bound transitions among the energy levels of ions embedded in the plasma which are considered in Chapters 3 and 4, or the free-bound transitions considered in Chapter 6. It is an obvious extension to apply the formalism to free-free transitions such as bremsstrahlung, though there the separation of radiator and plasma (or system and bath, to use the terminology of relaxation theory) is not straightforward.

If Stark broadening theory is thought of as addressing the question of how the plasma environment effects the process of radiative decay, then its techniques may be applied to other processes, such as radiative and collisional ionization and capture, dielectronic recombination and autoionization. These processes are important in establishing the population kinetics of the electronic energy levels of the radiating plasma ions. These detailed rates are important inputs into codes which calculate non-local thermodynamic equilibrium populations³³. The rates that are currently being used in such codes are isolated atom rates which do not account for any effects of the plasma environment on the relevant processes. The techniques and approximations developed

here should be useful if the effects of the plasma environment are to be approximated. As discussed in Chapter 6, it is possible that the vexing question of continuum lowering may also be addressed using the method introduced here.

X-ray line broadening spectroscopy is a mature and highly refined method of investigation into the extreme conditions in the core of laser-driven implosions. It will continue to serve both as an important practical tool, and an arena in which scientists can observe and work on complex, interesting plasma induced effects.

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BIOGRAPHICAL SKETCH

Donald Arthur Haynes, Jr., was born on April 11, 1966, in Elmhurst, New York, the firstborn child of Donald and Mary Haynes. Soon before his only sibling, Matthew, was born, the family moved to Bethpage, Long Island. He benefited immensely from the excellent public education available on Long Island at that time, and felt the first spark of his love of science when given the opportunity to run his middle school's planetarium. In 1980, the family moved to Jupiter, Florida, and what the educational system lacked was made up for by Donald's and his parents' insistence that he not be held back until the Jupiter school system finished teaching the tenth graders what the eighth graders in Bethpage were expected to know. It was in Jupiter, through the good offices of Mr. James Hughes and Mr. James Hilliker, that Donald got the opportunity to participate in dramatic and choral endeavors, and find the necessary counter-weights for the life of a scientist.

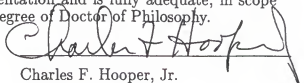
Donald attended the University of Miami from 1984 to 1988, receiving degrees in Physics and Mathematics. He joined the Alpha Sigma Phi social fraternity, from which society's open motto the imprint of this work "*Causa latet, vis est notissima.*" ("The cause is hidden, the results well known.") is taken. It was at the University of Miami that Donald met Karen Patricia Snyder, the woman who would do him the honor of becoming his wife.

Donald then began his graduate work in physics at the University of Florida. In 1990, he started this work, and plans to continue it throughout his profes-

sional career. In 1995, he entered into his most challenging endeavor, fatherhood, which he looks forward to pursuing throughout his life.

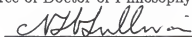
Any cogent analysis of Donald's life would have to conclude that he is an extremely fortunate man, fortunate to be born to parents of immense wisdom and kindness, fortunate to be the brother of a man whose oft-expressed senses of humor and irony belie his fundamental compassion, and fortunate to have the honor of being married to Karen, a woman whose patience is outweighed only by her ability to love.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



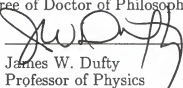
Charles F. Hooper, Jr.
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Neil Sullivan
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



James W. Dufty
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



John Klauder
Professor of Physics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.



Martin Vala
Professor of Chemistry

This dissertation was submitted to the Graduate Faculty of the Department of Physics in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1997

Dean, Graduate school